Self-Organized Traffic Flow and Human Coordination in Space and Time

Chair of Sociology, in particular of Modeling and Simulation

Exercises, Sheet 2

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1. Show by differentiation that (3.11) solves the wave equation (3.8). Moreover, check that formula (3.19) constitutes a special solution of the wave equation (3.8) which is compatible with the initial conditions $\delta\rho(x, 0)$ and $\partial\delta\rho(x, 0)/\partial t$. Why do we need two initial conditions (for the density and its derivative) to have a unique specification? Determine the special solution in the case (a) $\delta\rho(x, 0) = \exp(-x^2)$, $\partial\delta\rho(x, 0)/\partial t = 0$ and (b) $\delta\rho(x, t) = 0$, $\partial\delta\rho(x, 0)/\partial t = \cos(x)$.

2. Differentiate Eq. (3.16) partially with respect to x and t, considering Eq. (3.17), and insert the resulting expressions into Eq. (3.14) in order to show that Eqs. (3.16), (3.17) are a special solution of the Lighthill-Whitham-Richards model. Show that this solution is also consistent with the initial condition $\rho(x, 0)$. Simplify the solution of the LWR model for the case $c(\rho) = c = \text{const}$. Check that Eqs. (3.18) and (3.19) result in the case of the triangular fundamental diagram (3.53).

3. Derive the speed of shock waves from the conservation of the number of vehicles, assuming the densities ρ_- and ρ_+ directly downstream and upstream of the shock front.