Self-Organized Traffic Flow and Human Coordination in Space and Time

Chair of Sociology, in particular of Modeling and Simulation

Exercises, Sheet 1

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1. In the simplest specification of the social-force model

$$\vec{f}_{ij}^{\mathrm{rep}}\big(\vec{d}_{ij}(t)\big) = -A_i \mathrm{e}^{-d_{ij}(t)/B_i}\vec{e}_{ij}(t),$$

try to describe how each of the two parameters A and B affects the interactions between pedestrians.

2. Restricting the social force model on its right-hand side to the relaxation term (i.e. the velocity adaptation term), we have

$$\frac{d\vec{v}_i(t)}{dt} = \frac{v_i^0\vec{e}_i^0 - \vec{v}_i(t)}{\tau}\,. \tag{1}$$

What is the dynamical, general solution of this linear ordinary differential equation?

3. Derive the continuity equation in the n-dimensional from the conservation of the number of elements, i.e. from the fact that the temporal change in the number of elements in an infinitesimal (hyper-)cube of volume Δx^n is given by the inflows minus the outflows through its surface.

4. Derive the n-dimensional continuity equation for the density

$$\rho(\vec{x},t) = \int_{\mathcal{V}} d^n x' \; s(\|\vec{x}' - \vec{x}\|) \sum_i \delta\Big(\vec{x}' - \vec{x}_i(t)\Big) \tag{2}$$

from the social force model, using the symmetric smoothing function $s(\|\vec{x}_i(t) - \vec{x}\|)$.

5. Start with the continuity equation

$$\frac{\partial\rho(\vec{x},t)}{\partial t} + \vec{\nabla}_{\vec{x}} \cdot \big[\rho(\vec{x},t)\vec{V}(\vec{x},t)\big] = 0 \tag{3}$$

for n dimensions and derive that the number of elements in a closed system of volume $\mathcal{V}$ is constant in time.

6. Construct a speed-density relationship $V_{\mathrm{e}}(\rho)$ with $dV_{\mathrm{e}}(\rho)/d\rho \leq 0$, for which the fundamental diagram has several maxima.