A Sequential Investment Game

Ryan Murphy

I. A KNOWN DM PROBLEM

Consider a decision maker (DM) who is endowed with 100 units of currency. The DM can invest any amount of his current holdings on a random event. The random event is binary (i.e. has two mutually exclusive outcomes: positive or negative). If the outcome of the event is positive, then whatever the DM has invested is doubled and returned to the DM. If the outcome of the event is negative, then whatever the DM has invested disappears. The decision environment is multistage and each of these investment/outcome stages is repeated K times. On each stage, the DM can invest any amount (between 0 and the total of their holdings).

The probability of a positive outcome, p, ranges between (0.5 and 1) exclusive, and is constant for the entire duration of K stages. The DM knows this probability. These types of environments have a positive expectation for the DM because p is strictly greater than 0.5. The earnings for the DM at the end of the decision task are whatever he has at the end of K stages, or nothing if he goes bankrupt before K stages.

Clearly the DM should invest some of his money as the expectation is positive. But investing too much could be bad. There is some ideal level of risk exposure for the DM. What investment policy maximizes the DM's expectations for a given p?

The above problem is solved. See the Kelly Criteria for more information on this.

II. EXTENSION

Now, consider an extension of this problem in the context of a game between N players. The goal of each of the players is to have the highest total at the end of K stages. The payoff for this game is simply the winner (the player with the highest total after K stages) earns 1 unit whereas the others earn nothing. Ties are broken randomly if there is more than 1 player with the highest total.

The random events for the different players are iid across the K stages. The players do not know what is happening to each other as the game is progressing. They know that each of them are playing the same game with the same p and K, but they don't know what outcomes the others

have experienced nor what they have invested on each stage. Players only find out each other's total at the end of K stages.

In this context, what investment strategy maximizes the probability of winning? What would a rational player do, given that he is playing against N-1 other rational players.

To start, let p = .6, K=20, and N = 2. This is now a game (e.g. the domain of game theory) and the solution (Nash Equilibrium) to this problem is unknown. A closed form solution likely does not exist, but simulation could give insights into this problem.

III. CONTACT

Ryan O. Murphy (rmurphy@ethz.ch)
Assistant Professor and Chair, Decision Theory and Behavioral Game Theory