ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Modeling and Simulating Social Systems with MATLAB

Lecture 9–Dynamics on Networks

Olivia Woolley, Tobias Kuhn, Dario Biasini and Dirk Helbing

Chair of Sociology, in particular of

Modeling and Simulation

Schedule of the course

Introduction to MATLAB	17.02.	
	24.02.	
	03.03.	Introduction to social-science modeling and simulations
	10.03.	
Working on projects (seminar thesis)	17.03.	
	24.03.	
	31.03.	
	07.04.	
	14.04.	
	05.05.	
	12.05.	Handing in seminar thesis and giving a presentation
	19.05.	
	26.05.	canceled

Schedule of the course

Introduction to MATLAB

17.02.
24.02.

Different ways of Representing space

03.03. **Dynamical Systems (no-space)**

10.03. **Cellular Automata (grid)**

Working on projects (seminar thesis)

17.03.
24.03.
31.03.
07.04.
14.04.
05.05.
12.05.
19.05.
26.05.

Networks (graphs)

Continuous Space (…)

Final presentation schedule

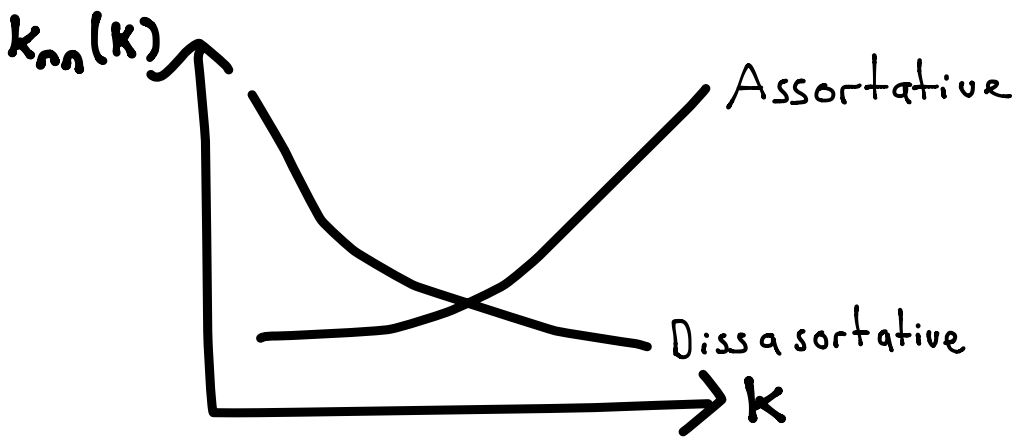
- Project presentation 15’ + 5’ (for Q&A)
- All group members have to actively participate in the presentation
- Registration for final presentation is binding; if you do not want to obtain credits, do **not** register!
- There are 18 slots on two days:
  - Wednesday, 14 May: 17:00 – 19:00
  - Thursday, 15 May: 17:00 – 19:00
  - Monday, 19 May: 17:00 – 19:00
- Sign up for slots begins **today**: http://goo.gl/4psqsM

I. Network Structure: Recap

- Canonical network topologies
- Paths and distance measures
- Measures of importance (centrality)
- Local structure
- Community structure

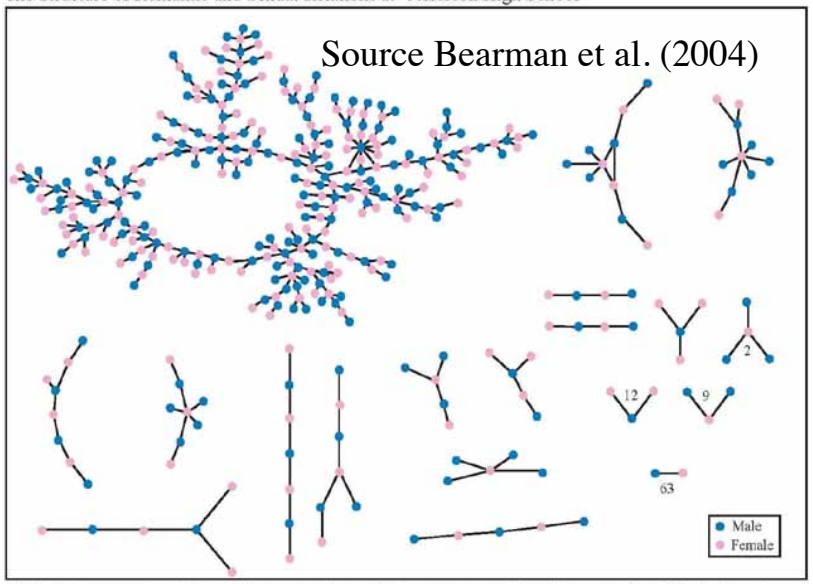
Network structure: Who connects to whom?

- Assortative mixing: Like attracts like.
- Can be any characteristic.
- E.g. Degree assortativity.
  - Average nearest-neighbor degree for vertices with degree *k*.



II. Disease spread on networks

The Structure of Romantic and Sexual Relations at "Jefferson High School"



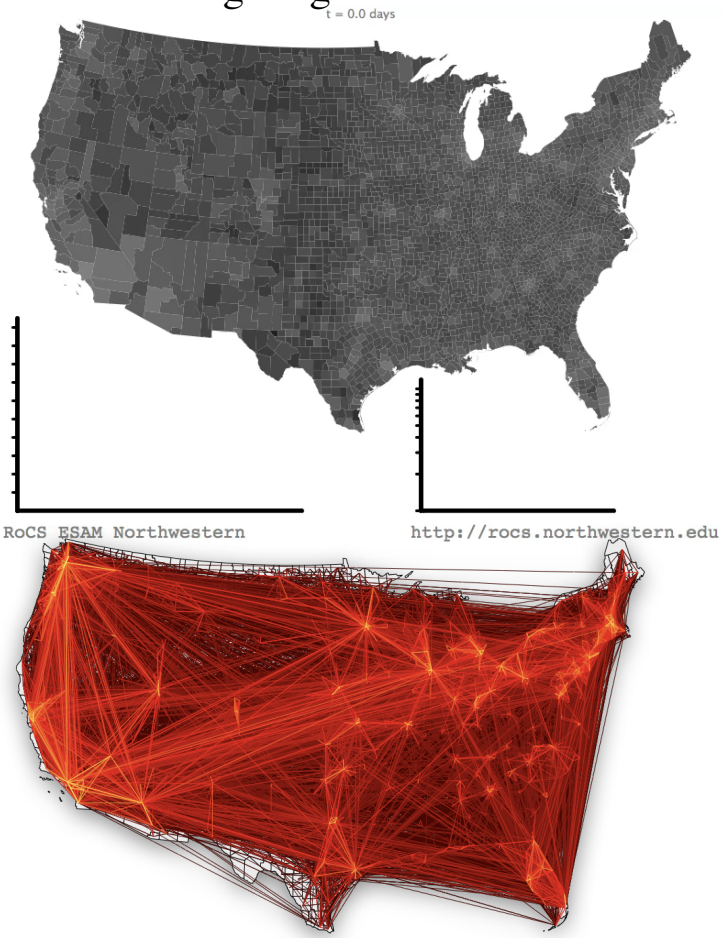
Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

- The structure of social interactions and human movement has a critical effect on disease spread.
- We can use networks to model this structure.

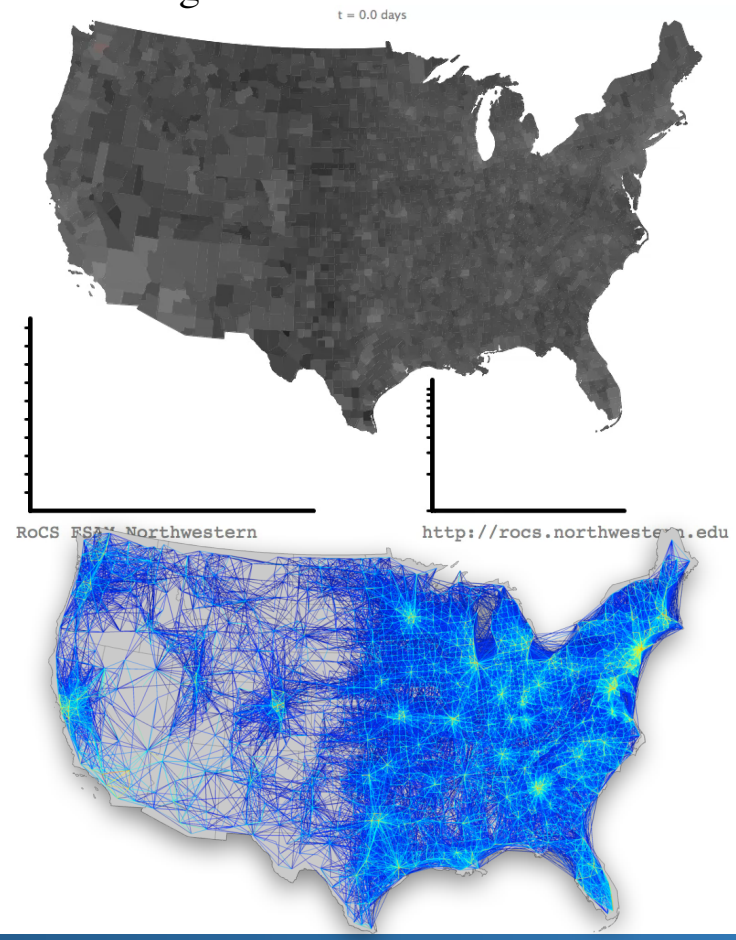
Effect of topology on disease spread

- Small diameter leads to faster spread.

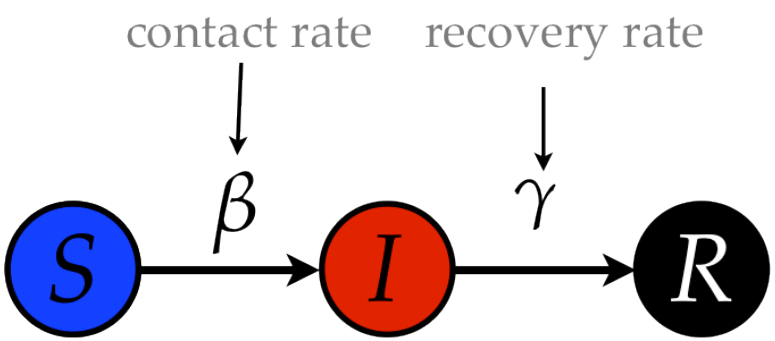
Long range connections

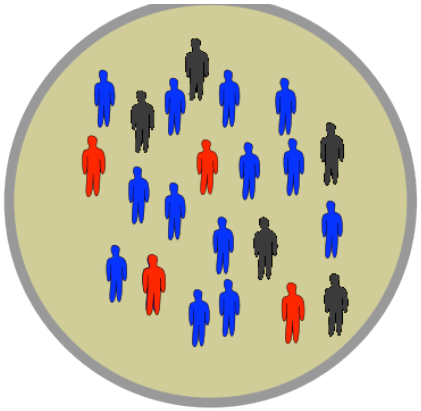


Short-range clustered connections



Recap: Kermack-McKendrick model





Susceptible Infected Recovered

Recap: Kermack-McKendrick model

S: Susceptible

I: Infected

R: Removed/recovered

β: Infection/contact rate

γ: Immunity/recovery rate

$$\frac{dS}{dt} = -\beta\, I(t)S(t)$$

$$\frac{dI}{dt} = \beta\, I(t)S(t) - \gamma\, I(t)$$

$$\frac{dR}{dt} = \gamma\, I(t)$$

Recap: Kermack-McKendrick model

N : Number of individuals
$s = S/N$
$j = I/N$
$r = R/N$

$$\frac{ds}{dt} = -\beta j(t)s(t)$$

$$\frac{dj}{dt} = \beta j(t)s(t) - \gamma j(t)$$

$$\frac{dr}{dt} = \gamma j(t)$$

Recap: Kermack-McKendrick model

N : Number of individuals
$s = S/N$
$j = I/N$
$r = R/N$

$$\frac{ds}{dt} = -\beta j(t)s(t)$$

$$\frac{dj}{dt} = \beta j(t)s(t) - \gamma j(t)$$

$$\frac{dr}{dt} = \gamma j(t)$$

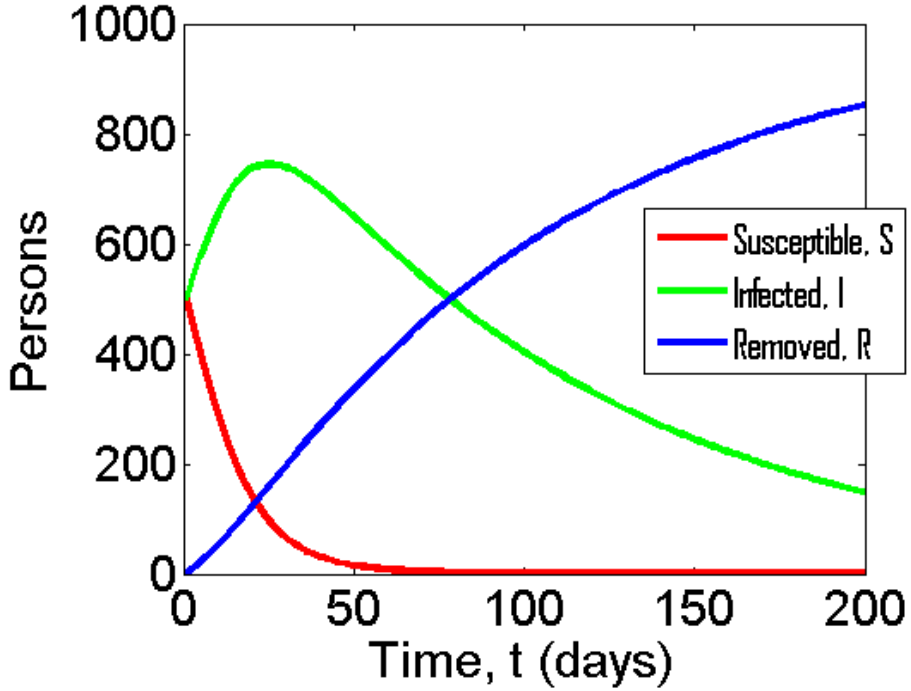
The disease will die out if:

$$\frac{dj}{dt} \leq 0 \Rightarrow \beta \geq \gamma$$

Recap: Reproductive number

$$R_0 = \frac{\beta}{\gamma}$$

$$R_0 > 1 \Rightarrow \text{ Infection invades population}$$



Approximating spreading on a network

- Assume that each of the *k* neighbors is equally likely to be of type Infected.
- Probability that a node with *k* neighbors becomes infected in time interval *dt*:

Expected number of infected neighbors

$$1-(1-\underbrace{\beta dt}_{\text{Probability contact occurs with a single infected neighbor}})^{\overbrace{kj}} \simeq kj\beta dt$$

(Leading order of Taylor expansion when *βdt* << *1)*

Approximating spreading on a network

N : Number of individuals

$s = S/N$

$j = I/N$

$r = R/N$

Average degree

$$\frac{ds}{dt} = -\beta \langle k \rangle j(t) s(t)$$

$$\frac{dj}{dt} = \beta \langle k \rangle j(t) s(t) - \gamma j(t)$$

$$\frac{dr}{dt} = \gamma j(t)$$

Approximating spreading on a network

N : Number of individuals
s = *S/N*
j = *I/N*
r = *R/N*

$$R_0^{\text{eff}} = \frac{\beta\langle k\rangle}{\gamma}$$

$$\frac{ds}{dt} = -\beta\langle k\rangle j(t)s(t)$$

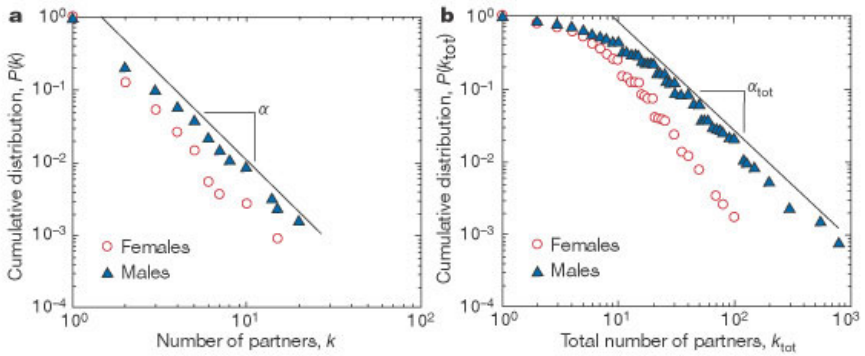
$$\frac{dj}{dt} = \beta\langle k\rangle j(t)s(t) - \gamma j(t)$$

$$\frac{dr}{dt} = \gamma j(t)$$

- Easier for disease to invade the population for larger <*k*>

Super-spreaders: Network heterogeneity

- What happens when the degree distribution is heterogeneous? (e.g. scale free)

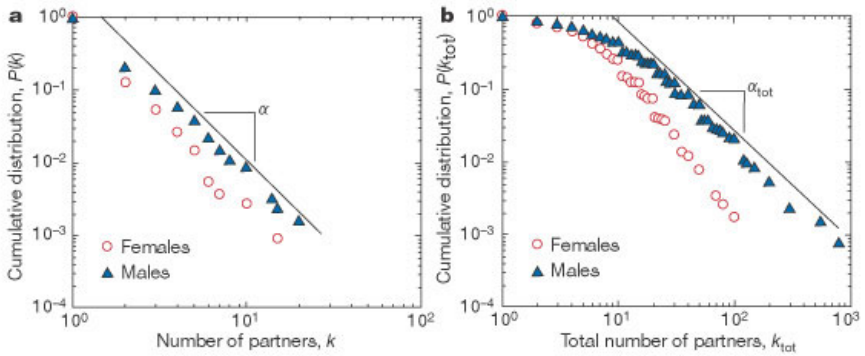


Source: Liljeros et al. (2001)

Swedish survey of sexual behaviour (1996)

Super-spreaders: Network heterogeneity

- What happens when the degree distribution is heterogeneous? (e.g. scale free)



Source: Liljeros et al. (2001)

Swedish survey of sexual behaviour (1996)

- Write down a model that explicitly tracks the state and degree of individuals.

Super-spreaders: Network heterogeneity

N : Number of individuals

$s = S/N$

$j = I/N$

$r = R/N$

density of infected neighbors around a node with degree k

$$\frac{ds_k}{dt} = -\beta k s_k(t)\Theta_k(t)$$

$$\frac{dj_k}{dt} = \beta k s_k(t)\Theta_k(t) - \gamma j_k(t)$$

$$\frac{dr_k}{dt} = \gamma j_k(t)$$

$$\Theta_k(t) = \Theta(t)$$

Super-spreaders: Network heterogeneity

$$R_0^{\text{eff}} = \frac{\beta \langle k \rangle}{\gamma} \quad \Rightarrow \quad R_0^{\text{eff}} = \frac{\beta}{\gamma} \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

For derivation see Pastor-Satorras et al. (2001) and Barrat et al. (2008)

- As heterogeneity increases invasion is more likely
- Epidemics always occur in scale free networks with a very broad degree distribution:

$$p(k) = Ck^{-\alpha} \text{ with } \alpha \leq 3 \implies \langle k^2 \rangle \to \infty \text{ as } N \to \infty$$

Exploiting structure for disease control

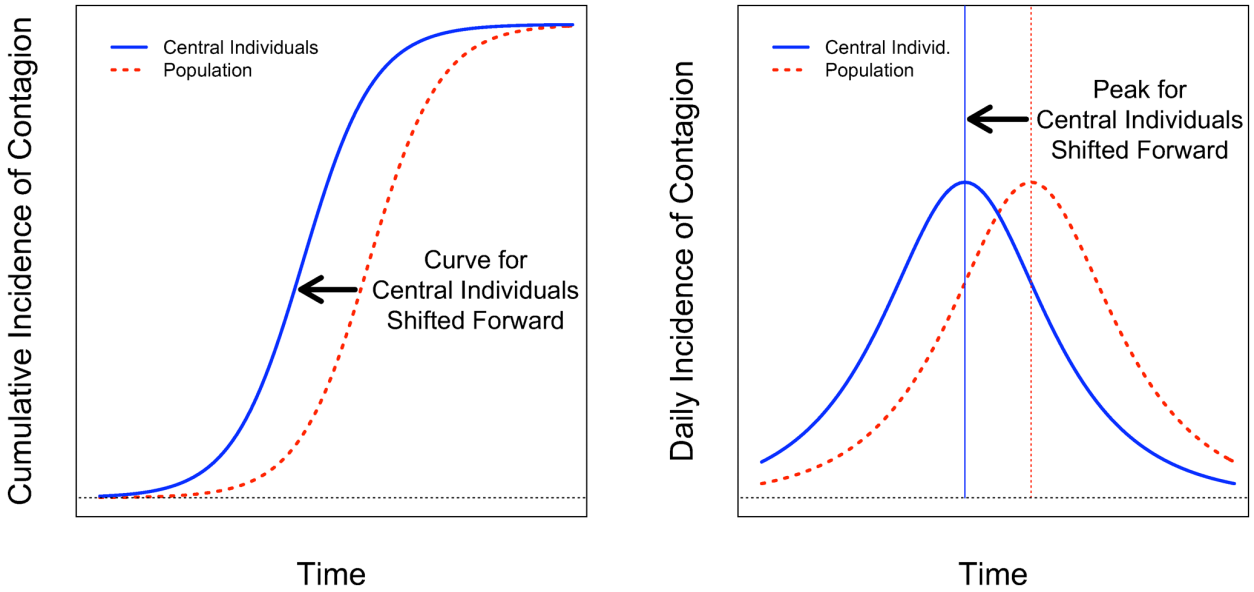
- Which nodes would you immunize to stop the spread of disease most effectively?

Exploiting structure for disease control

- Which nodes would you immunize to stop the spread of disease most effectively?
  - More central nodes! (e.g. higher degree)

Exploiting structure for disease control

- Which nodes would you immunize to stop the spread of disease most effectively?
  - More central nodes! (e.g. higher degree)



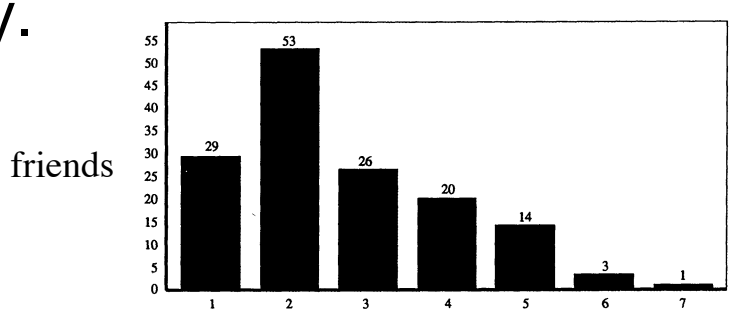
Source: Christakis et al. (2010)

Exploiting structure for disease control

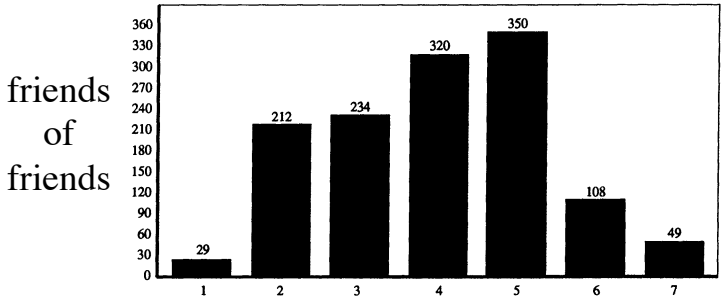
- Which nodes would you immunize to stop the spread of disease most effectively?
  - More central nodes! (e.g. higher degree)
- To do this we need a lot of information about the network structure. Usually not feasible.
- Smart local solution based on the way you sample individuals in a network.

Your friends have more friends than you

- The average degree of nearest neighbors is larger than the average degree.
  - Intuition: Higher degree nodes are counted more frequently.



a) The mean is 2.7.



Source: Feld (1991)

Your friends have more friends than you

- The probability that a randomly chosen node has degree k':

Number of nodes with degree k

$$P(k') = \frac{N_{k'}}{N}$$

Number of nodes

- Probability that a neighbor has degree *k*':

$$\frac{k' N_{k'}}{\sum_{k''} k'' N_{k''}} = \frac{k' N_{k'}}{\langle k \rangle N} = \frac{k'}{\langle k \rangle} P(k')$$

Average degree

(assuming neighbor degrees are not correlated)

Exploiting structure for disease control

- Which nodes would you immunize to stop the spread of disease most effectively?

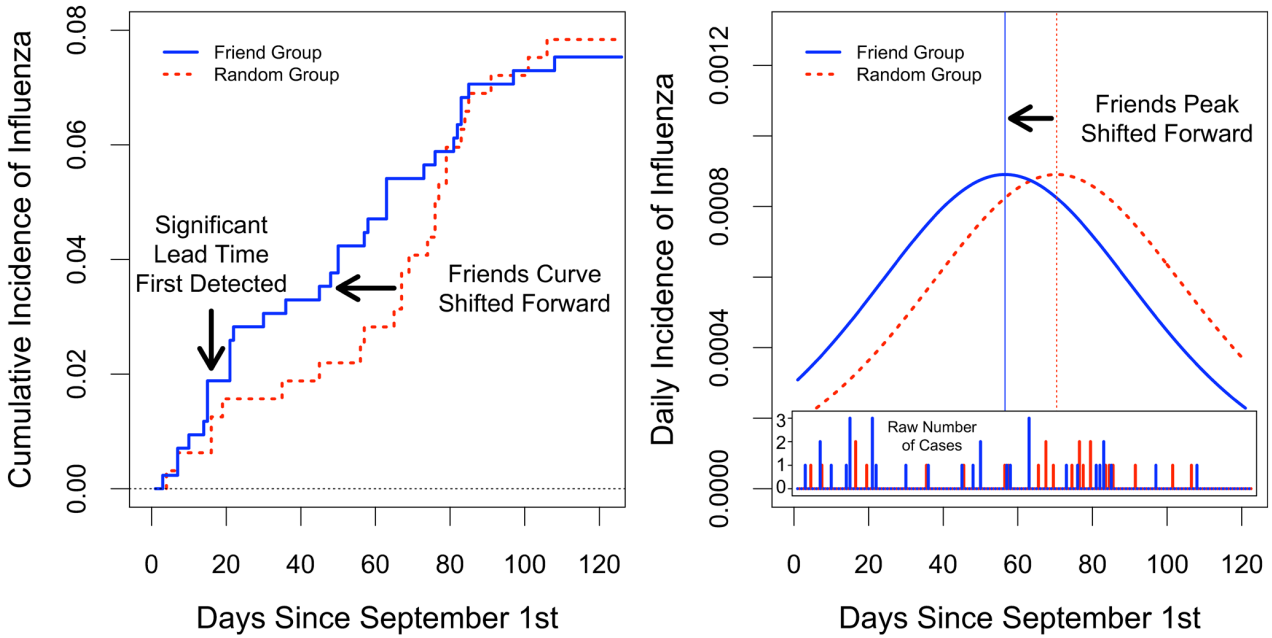
Exploiting structure for disease control

- Which nodes would you immunize to stop the spread of disease most effectively?
  - The friends of randomly chosen individuals.

Exploiting structure for disease control

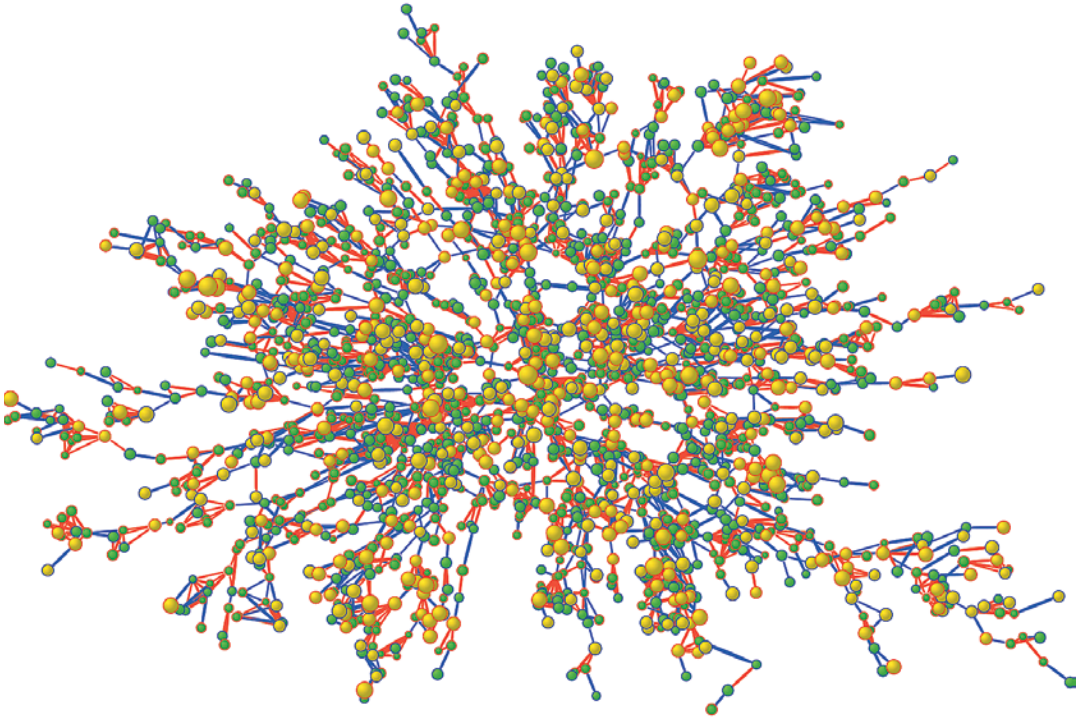
- Which nodes would you immunize to stop the spread of disease most effectively?
  - The friends of randomly chosen individuals.

Source: Christakis et al. (2010)



Social sensors for early warning

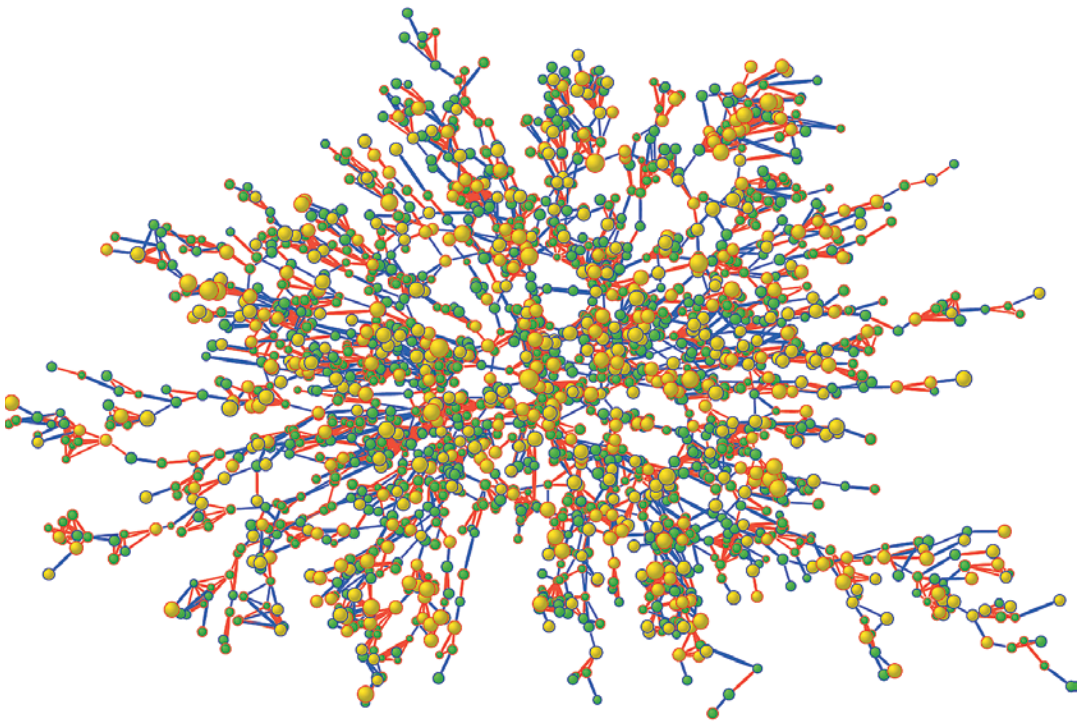
Models of social spreading



Source: Christakis et al. (2007)

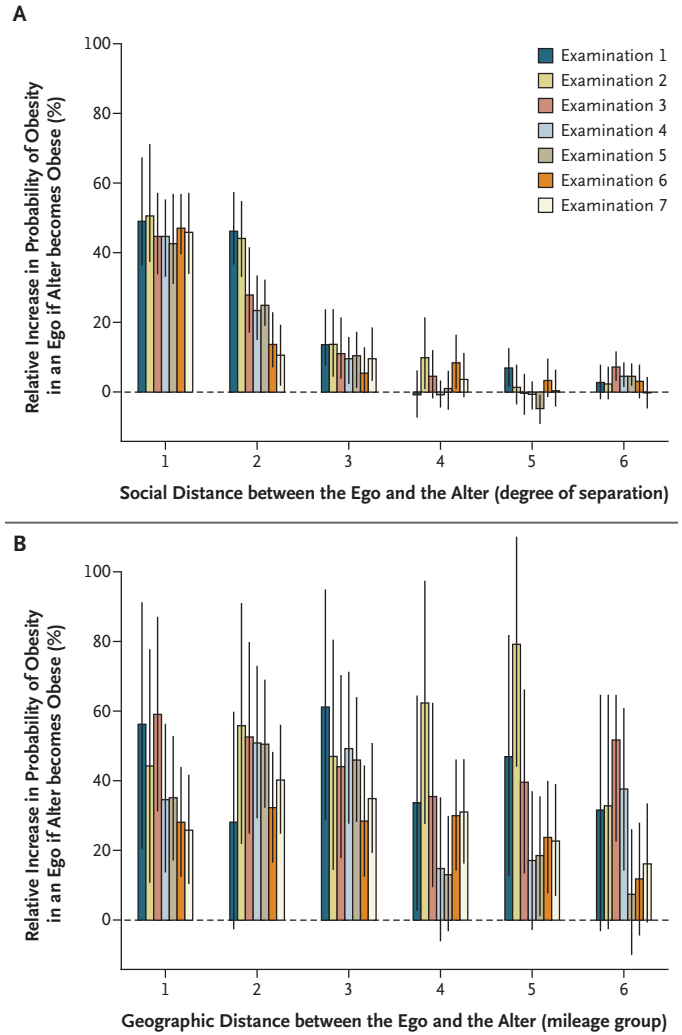
Can your friends make you fat?

Models of social spreading



Source: Christakis et al. (2007)

Can your friends make you fat?



Is it really influence?

Observation: You are more likely to be fat if you have fat friends.

- Three competing hypothesis:
  - Social influence
  - Homophily
  - Covariation of another variable

Is it really influence?

Observation: You are more likely to be fat if you have fat friends.

- Three competing hypothesis:
  - Social influence: Behavior spreads from one friend to another. You like McDonalds and because of this I start liking it too.
  - Homophily
  - Covariation of another variable

Is it really influence?

Observation: You are more likely to be fat if you have fat friends.

- Three competing hypothesis:
  - Social influence
  - Homophily: Similar people are more likely to be friends. We both like McDonalds so we're more likely to meet or like each other.
  - Covariation of another variable

Is it really influence?

Observation: You are more likely to be fat if you have fat friends.

- Three competing hypothesis:
  - Social influence
  - Homophily
  - Covariation of another variable : We are friends because we live in the same neighborhood and there are many McDonalds there.

Is it really influence?

Observation: You are more likely to be fat if you have fat friends.

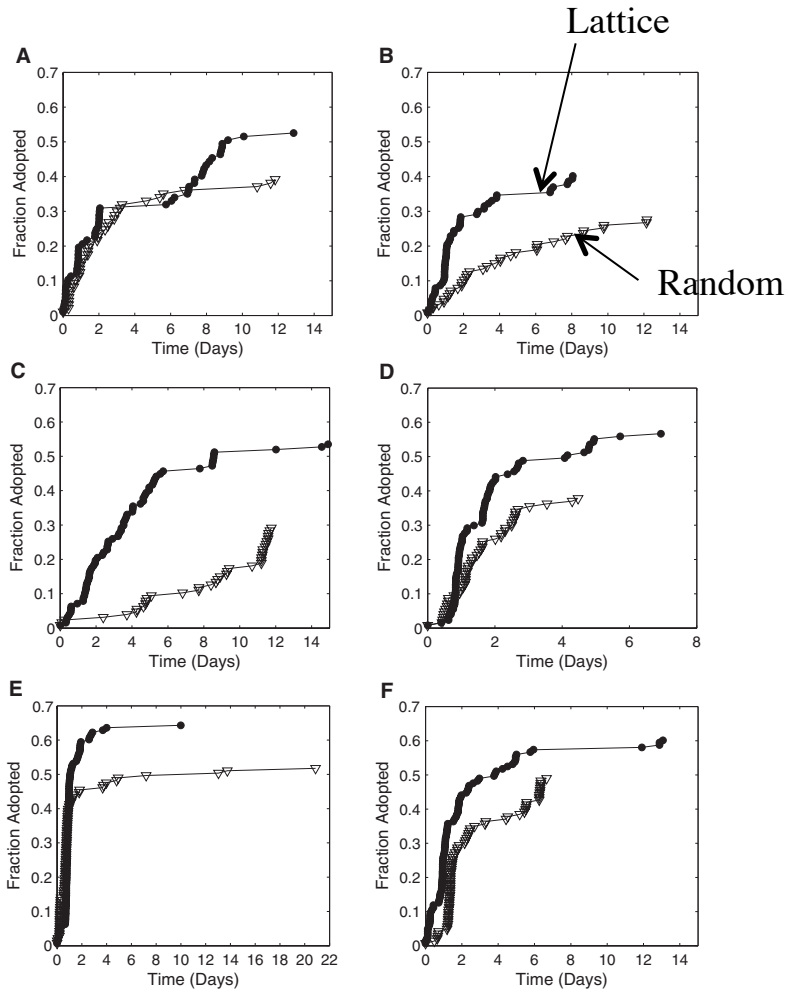
- Three competing hypothesis:
  - Social influence: Behavior spreads from one friend to another. You like McDonalds and I like to eat with you.
  - Homophily: Similar people are more likely to be friends. We both like McDonalds so I think you're cool.
  - Covariation of another variable : We are friends because we live close and there are many McDonalds in our neighborhood.

Impossible to distinguish hypothesis without a controlled experiment!

(See Shalizi et al. 2011).

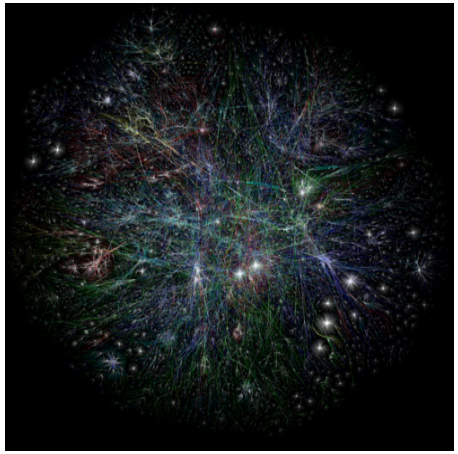
Complex contagion

- Assume that “infection” (or adoption) requires multiple “infective” contacts.
- Spreading can be faster with short-range connections!

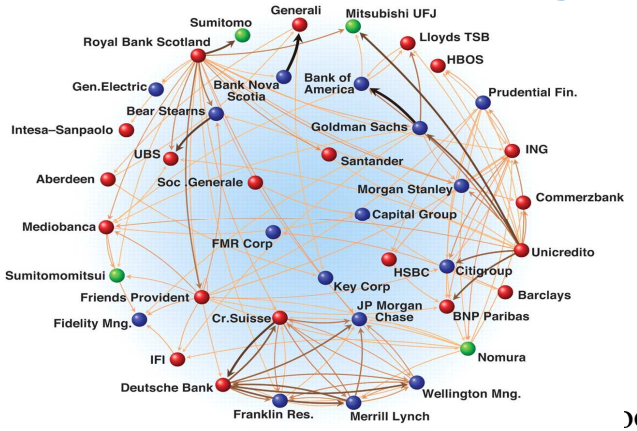


Source: Centola (2010)

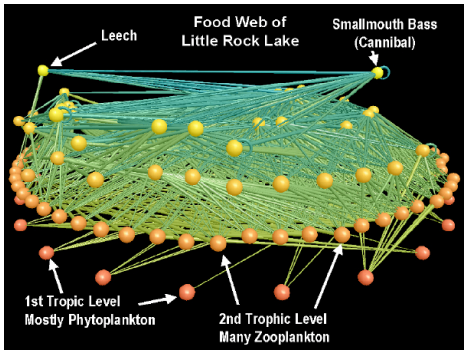
III. Resilience of Network Systems



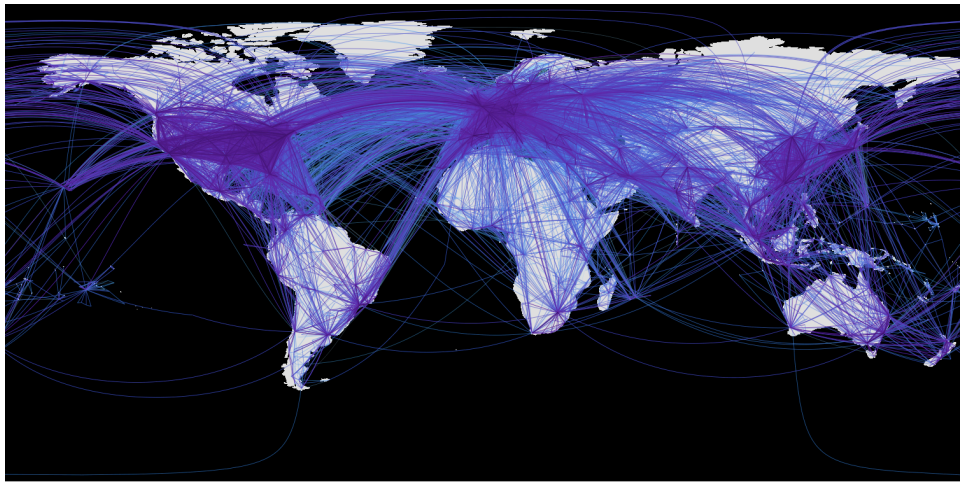
Internet [opte project]



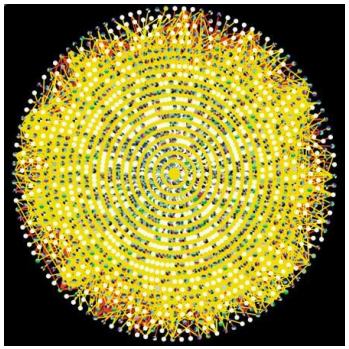
Banking Network



ood Webs and ecosystems [Martinez '91]



Global air travel



Protein Interactions [genomebiology.com]

Resilience of Network Systems

How does damage at a small fraction of network components influence the functionality of the entire system?

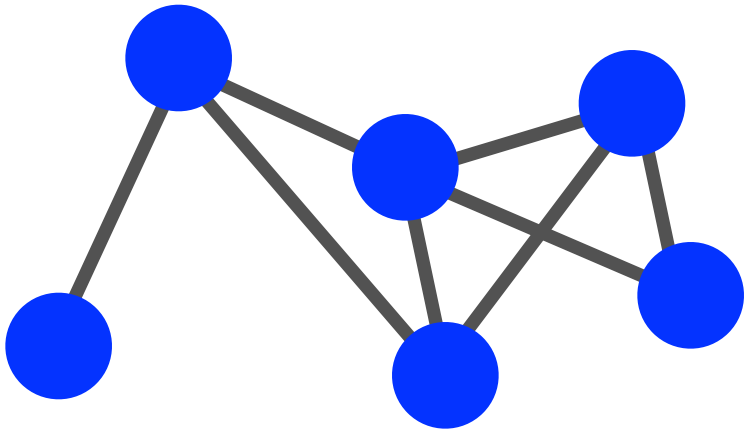
Define measures of the impairment to proper function

- Fragmentation

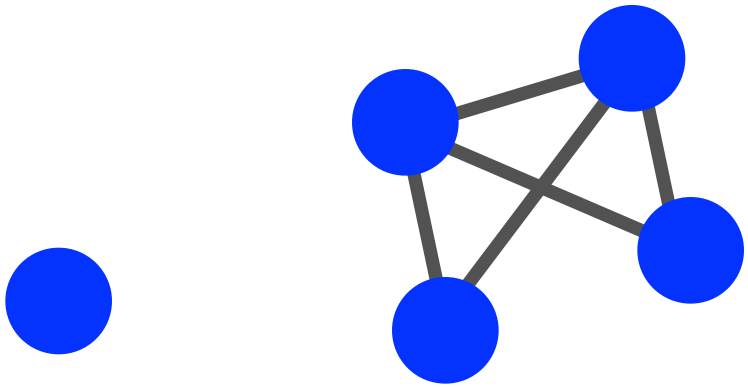
Evaluate impact under different classes of perturbations

- Random failure
- Targeted attacks removing nodes ranked by centrality

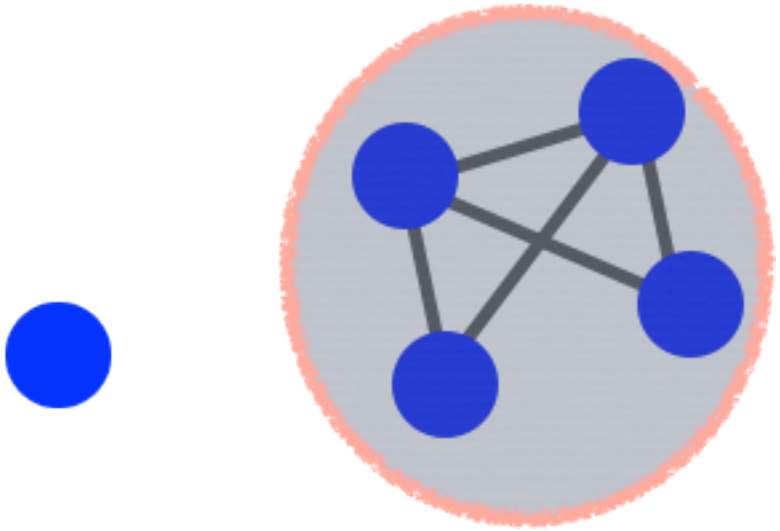
Fragmentation



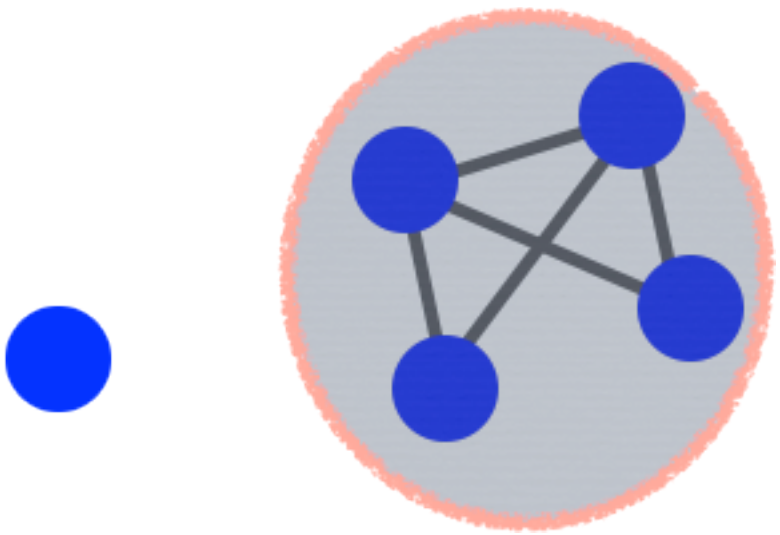
Fragmentation

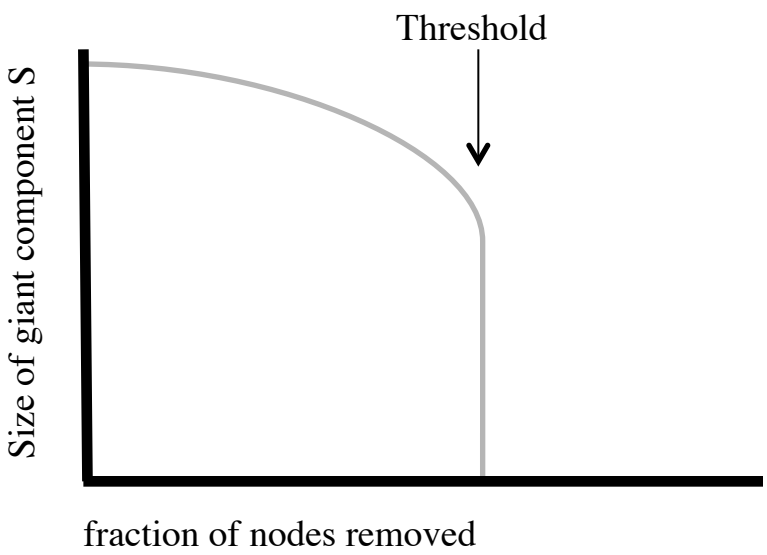


Fragmentation



Fragmentation

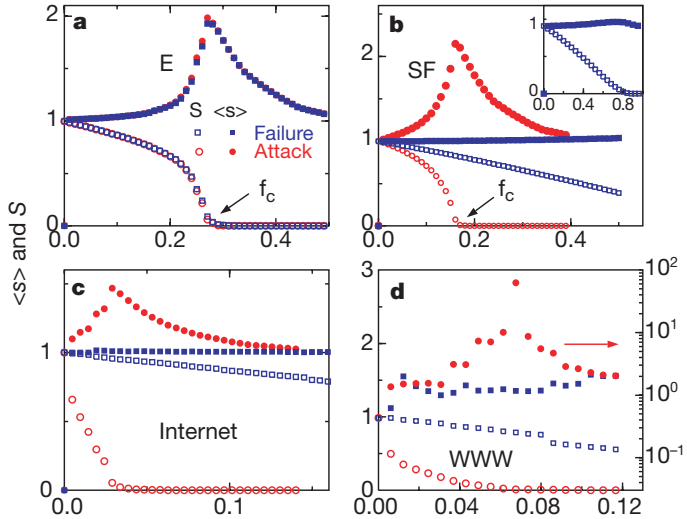




- The network breaks into disconnected components.
- Analogous to the percolation problem we studied to understand the emergence of a giant component.

Topology and resilience

- Random networks:
  - Equal effect due to random failure and targeted attacks
- Networks with scale free degree distribution:
  - Resilient to random failure
  - Vulnerable to targeted attacks



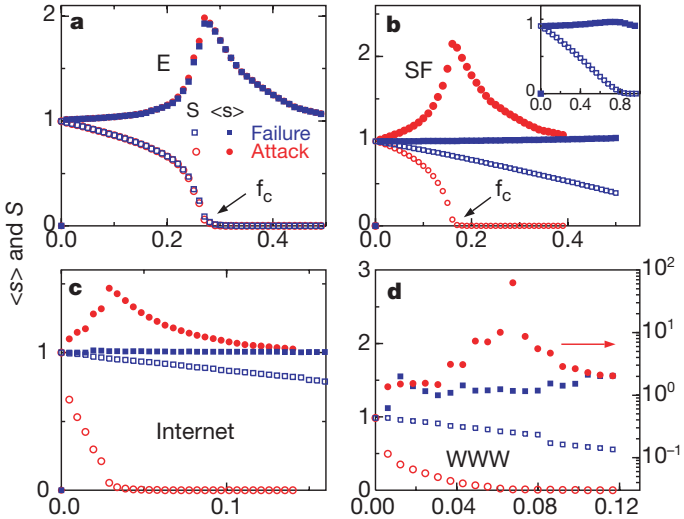
Source: Albert et al. (2000)

Open symbols denote S

Topology and resilience

- Random networks:
  - Equal effect due to random failure and targeted attacks
- Networks with scale free degree distribution:
  - Resilient to random failure
  - Vulnerable to targeted attacks

Hypothesis:
Scale free networks are common because they are more resilient?



Source: Albert et al. (2000)

Open symbols denote S

References

- Barrat, A., Barthelemy, M., & Vespignani, A. (2008). *Dynamical processes on complex networks*. Cambridge: Cambridge University Press.
- Feld, S. L. (1991). Why your friends have more friends than you do. *American Journal of Sociology*, 1464-1477.
- Bearman, P. S., Moody, J., & Stovel, K. (2004). Chains of Affection: The Structure of Adolescent Romantic and Sexual Networks1. *American Journal of Sociology*, *110*(1), 44-91.
- Liljeros, F., Edling, C. R., Amaral, L. A. N., Stanley, H. E., & Åberg, Y. (2001). The web of human sexual contacts. *Nature*, *411*(6840), 907-908.
- Pastor-Satorras, R., & Vespignani, A. (2001). Epidemic spreading in scale-free networks. *Physical review letters*, *86*(14), 3200.
- Christakis, N. A., & Fowler, J. H. (2010). Social network sensors for early detection of contagious outbreaks. *PloS one*, *5*(9), e12948.

References Continued

- Christakis, N. A., & Fowler, J. H. (2007). The spread of obesity in a large social network over 32 years. *New England journal of medicine*, *357*(4), 370-379.
- Shalizi, C. R., & Thomas, A. C. (2011). Homophily and contagion are generically confounded in observational social network studies. *Sociological Methods & Research*, *40*(2), 211-239.
- Damon Centola, The Spread of Behavior in an Online Social Network Experiment. Science, *Vol. 329 no. 5996 pp. 1194-1197 (2010)*
- Albert, R., Jeong, H., & Barabási, A. L. (2000). Error and attack tolerance of complex networks. *Nature*, *406*(6794), 378-382.

Projects

- Today, there are no exercises. Instead, you can work on your projects and we will supervise you.