## ETH

# Lecture with Computer Exercises: Modelling and Simulating Social Systems with MATLAB 

Project Report

## Fuel Consumption on a single lane highway under variable traffic densities.

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## 1. Individual contributions

We both were involved in every aspect of the project as most of the time we worked together. Furthermore we split the work in an equivalent way, meaning that both computed results as well as code. In order to write down the corresponding report we split the work in two equal part.

## 2.Introduction and Motivation

During the last fifteen years much has changed in the world. People more and more have started to care about the climate, especially the global warming and its follow-ups. As a consequence, improvements can be seen in the car industry. More cars are produced to use less fuel or to use fuels which do reduce the $\mathrm{CO}_{2}$ emission.

Furthermore petrol is getting really expensive[4] such that carrier and even normal family try to reduce the fuel consumption and therefore their costs.

Such example shows the importance of minimizing fuel consumption. The question is now: How is one able to minimize fuel consumption? The purpose of this work is to give an insight on the fuel consumption and answer this question by simulating traffic on a single-lane highway under few circumstances.

Analysis will be focused on the traffic density and the maximum velocity of a small and a large car. The authors wanted especially to show with their simulation, that it is better to transport things under very low densities than during the rush hour. Furthermore, impact of drivers' behaviour will be investigated to prove that rational behaviour is better in terms of consumption. Respective effects of maximum velocity and traffic density on fuel consumption will finally be analysed.

The density at which traffic jam start to occur will also be defined. This density is important to be known in order to reduce shipment costs.

## 3.Description of the Model

In this chapter the model is explained with which we simulate the fuel consumption at different maximum velocities and different densities. The latter can be split up in two different sub models, the highway simulation and the simulation of the fuel consumption. The highway simulation calculates the velocities at given densities and maximum velocities and then with the fuel consumption simulation we can compute the consumption of the car of interest.

### 3.1 The Highway simulation

Iterations of cars' velocity are executed according to the highway the Nagel-Schreckenberg-Model. This model was introduced in the early 1990's by Kai Nagel and Michael Schreckenberg. It is one of the first models which described the traffic and is able to model traffic jams under quite simple assumptions. As a drawback with respect to simplicity, drivers behaviour as well as road characteristics are described in a rather simple way. [1]

The Nagel-Schreckenberg-Model is a cellular automaton, that means that there are cells of a length of C in which there is one car or there is not. It is built up out of three essential assumptions:
(1) Cars do always accelerate, until it gets to a maximum velocity. How fast it accelerates depends on the conditions but it still has to be natural number of cells. Otherwise the car would not fit in only one cell.
(2) If the car is near to another car, that means the gap between both cars is smaller than the velocity of the car behind, the latter is set to this gap.
(3) To add some mistakes of drivers e.g. concentration mistakes, there is a probability, that the car decelerates by some natural number of cells. This probability is called the dawdling probability.

In every time step T the velocity is calculated with respect to these assumptions and the cars are moved to their next cell.

In order to be able to iterate cars over the highway, a rather strong assumption needs to be formulated. It accounts for the fact that knowing speed and velocity is enough to know car's displacement.

The drawback is as follows: Either a cell length can be assumed to be equal to a characteristic length of car, which leads to lose of space resolution, as cars are not allowed to move forward for less than a cell. The second possibility is to set the cell length to unity. Therefore cars can drive from cell to cell, but car characteristic length is rather wrong.

As a drawback the second solution was chosen as we need a small length resolution within traffic jam and furthermore cars length are not really important.

In the end we have one last thing to assume: the acceleration in (1) and the deceleration in (3) is $1 \mathrm{~ms}^{-2}$. This assumption comes from the feeling, that a normal driver does not accelerate with the full power of the car.

So to sum up everything the velocity $v_{\text {car }}(t)$ is recursively defined to be:

$$
v_{c a r}(t+1)= \begin{cases}v_{c a r}(t)+1, & v_{c a r}(t)<\text { gap; no dawdle }  \tag{2.1}\\ v_{c a r}(t) & , \quad v_{c a r}(t)<\text { gap } ; \text { dawdle } \\ \text { gap } & , v_{c a r}(t) \geq \text { gap; no dawdle } \\ \text { gap }-1, & v_{\text {car }}(t) \geq \text { gap; dawdle }\end{cases}
$$

As initial velocity is chosen randomly for every car the new velocity can be defined for every time step according to statements (2.1). To see how have the velocity iterations been computed, refer to Nagel-Schreckenberg function partially described in figure 3.

### 3.2 Fuel consumption

In order to be dimensionally consistent, every value should be calculated in S.I. units. Simulation of the fuel consumption is implemented in the fuelconsumption function. Energy consumption of the car is calculated in every cell within which the car is going through. As velocities are constants over each time step and actualized instantaneously they are always constants over a given number of cell. The simulation of the fuel consumption is based on the simple idea, that the total energy used is accounting for the kinetic energy $\Delta E_{\text {kin }}$, the friction energy $E_{f r i c}$ and the so-called threshold energy term $E_{\text {const }}$ used just by running the motor (this term is the one which uses most of the fuel in traffic jams).

$$
\begin{equation*}
\Delta E_{\text {tot }}=\Delta E_{\text {kin }}+E_{\text {fric }}+E_{\text {const }} \tag{2.2}
\end{equation*}
$$

In this formula is also assumed, that the potential energy is zero, therefore cars are moving on a plane street. The friction energy is given by two terms, the friction of the tyres $E_{t f}$ and the air resistance $E_{a r}$.

For the air resistance we used firstly Stoke's law (2.6) by assuming that the car is a sphere. As it did not provide the expected results we turned on the equation of the air friction in order to match more suitably the energy consumption. [5]

$$
\begin{gather*}
E_{f r i c}=E_{t f}+E_{a r}  \tag{2.3}\\
E_{t f}=f m g l  \tag{2.4}\\
E_{a r}=\frac{1}{2} c_{w} \rho_{a i r} A v^{2} l  \tag{2.5}\\
E_{\text {stokes }}=6 \pi r \eta_{a} v l \tag{2.6}
\end{gather*}
$$

Here $f$ is the friction coefficient, $l$ the length with constant velocity, $\eta_{a}$ the viscosity of air, $r$ the radius of the car, $c_{w}$ the air friction coefficient, $\rho_{\text {air }}$ the density of air, $A$ the projection area of the car in the driving direction and $v$ is the velocity. We can calculate I directly with

$$
\begin{equation*}
l=v \Delta t \tag{2.7}
\end{equation*}
$$

For the kinetic energy there are two cases to classify, when the drivers breaks and when he accelerates. We assume that our car is not a hybrid, so the energy while breaking down will just be transformed into heat. In the other case it is just the normal formula for the kinetic energy:

$$
\Delta E_{k i n}= \begin{cases}\frac{1}{2} m(\Delta v)^{2}, & \Delta v \geq 0  \tag{2.8}\\ 0 & \Delta v<0\end{cases}
$$

With these equations we defined the total energy at every time step. To get now to the fuel consumption we need to know the engine efficiency $\eta_{e}$, the heating value $\Delta_{r} H$ and finally the density $\rho$ of our petrol. Knowing these parameters the volume needed to provide enough energy can be calculate the volume. To be consistent with the velocity we use always SI-units for the parameters.

$$
\begin{equation*}
E_{t o t}=\Delta_{r} H \rho \eta_{e} \Delta V \Rightarrow \Delta V=\frac{E_{t o t}}{\Delta_{r} H \rho \eta_{e}} \tag{2.9}
\end{equation*}
$$

For this purpose we used diesel as petrol, which has a large heating value of $45 \mathrm{MJ} / \mathrm{kg}$ [3]. Using the equations (2.1) - (2.9) the final volume is given by (2.10) and then can be easily calculated in terms of liter.

$$
\begin{equation*}
\Delta V=\frac{\Delta E_{k i n}+f m g v \Delta t+\frac{1}{2} c_{w} \rho_{\text {air }} A v^{3} \Delta t+E_{\text {const }}}{\Delta_{r} H \rho \eta_{e}} \tag{2.10}
\end{equation*}
$$

## 4.Implementation

First of all we intend to implement a model that could give an idea of the fuel consumption of two categories of cars on a flat high way long of 100 km varying the maximal authorized velocity as well as the traffic density.

As seen in 3.1 the following assumptions are useful in the implementation step:

- Single lane highway.
- Drivers are respectful of the maximum authorized velocity, which means that they don't drive faster than the latter.
- Cells are 1 m long.
- Acceleration is constant over each cell.
- Cars can either not accelerate, accelerate of $1 \mathrm{~m} / \mathrm{s}$ or decrease their speed in order to avoid collision with the car ahead.
- Cars are spread randomly over the highway. The first cell corresponds is the starting place of the interesting car.

With the latter assumptions, a simulation is possible using the Nagel-Schreckenberg model in order to iterate the velocity. The car of interest is set on the first cell at rest as the others are spread with a random initial velocity on the highway. Once they reached the end of the highway, they are simply deleted.
The algorithm will be split in the following subsections in order to give additional explanations:

- Initialisation of cars on the highway.
- Iterations over time.
- Nagel-Schreckenberg function.
- Plot and result section
- Fuel consumption function.

In order to spread the cars randomly on the highway, the following algorithm was used:

```
%Initialisation of the other cars on the highway, their are spread randomly
%on a free space between x=0 and x=L.
n = 2;
while (n <= n_cars-1)
    initial_place = ceil(abs(ceil(rand(1)*L)));
    if ( initial_place <= L )
            %If there is a free space (car()=0) we can put a car! Either not!
            if ( car(initial_place) == 0)
            car(initial_place) = 1;
                speed(initial_place) = abs(ceil(rand(1)*vmax));
                n = n + 1;
            end
    end
end
```

Figure 1: Initialisation of the other cars.

Here $n$ is an iteration variable ranging from 2 to the number of cars. The number of cars is simply the density times the length of the highway. The rand(1) Matlab function permits to have numbers uniformly distributed between 0 and 1. Ceil Matlab function rounds the number upwards.
If a car can be set on a corresponding cell (free cell), an initial velocity needs to be calculated as a function of the maximal velocity authorized on the highway.
If a car is already present on the corresponding cell we recalculate an initial place as well as an initial speed until a free space is found.
Once every car has been distributed on the highway, we can move further and start to iterate their corresponding emplacements and velocities.

The part of the code used to update the position and the speed of every single car is called Iterations over time and is depicted below:

```
%Iterations over time:
i=1;
while (mod(x,L) ~=0 &&& i<n_iterations)
    car_t_plus_1 = zeros(1,L);
    speed_plus_1 = zeros(1,L);
    %============================================================================
        for cell_number = 1:L
            if (mod(cell_number,L) ~= 0)
                if (car(cell_number) == 1)
                                    %---------------------------------------------------------
                                    v=nagel_schreckenberg (cell_number, speed(cell_number), car);
                                    %---------------------------------------------------------------
                                    if (x==cell_number)
                                    vt (i+1)= v;
                                    x=x+v;
                                    end
                                    car_t_plus_1(cell_number+v) = 1;
                                    speed_t_plus_1(cell_number+v) = v;
                end
            end
        end
    car = car_t_plus_1;
    speed = speed_t_plus_1;
    i=i+1;
end
```

Figure 2: Iterations over time.
The goal is here to permit the car to drive through the highway. The purple box including the while statement permits the cars to reach the end. A control parameter is included in order to avoid iteration towards infinity if for one reason or another, the velocity tended to 0 for a while. When a car is present on a given cell, the latter is described by the value 1 in the car vector; instead if no car is present, a zero is set.

The red box is calling an auxiliary function called nagel_schreckenberg which implements the new velocity of the car by means of the cell number and the speed of the car on the actual cell. Car is also needed and represents the vector that helps us to determine if there is a car or not on the corresponding cell.

The green box denotes the additional statement in order to obtain the displacement of the interesting car.

The new position of the cars as well as their corresponding velocity is recorded into car and speed vectors.

The Nagel-Schreckenberg function is depicted below:


Figure 3: Nagel-Schreckenberg function
This function is computing the new velocity as a function of the previous one as well as the previous position of the car and the one of the next car ahead. (hint: acceleration are assumed to be instantaneous).

The red box denotes the default acceleration which is assumed for every car to be independent of the actual position.

The green box is calculating the minimum gap in order to avoid collision with the car ahead, the purple box adapts the speed of the car according to this latter gap. Finally the hell blue box stands for the probability of keeping a constant speed where rand denotes a random number uniformly distributed between 0 and 1 .

The plot and result section calculates the parameters we want to investigate by means of a fuelconsumption function which returns the energy as well as the fuel consumed per unit cell.

This function is described by the following terms:

```
if(vt(i)-vt(i-1))>0
    Ekin=(1/2)*w_large*(vt(i)-vt(i-1))^2;
else
    Ekin=0;
end
E=density_air*A_large*cw/2*vt(i)^3+friction_large*g*vt(i)*w_large+Ekin+Econst;
F=E/ (heat_power*efficiency*density_diesel);
```

Figure 4: Important part of the fuel consumption function.
The red box is depicted in figure 4 represents the kinetic energy loss if the car decelerates. If the car decelerates, the kinetic energy remains constant accounting for the loss of energy. The green box is computing the energy used in each cell as well as the fuel consumed in each cell.
The parameters used in order to calculate the energy are given in table $\mathbf{2}$ in the Simulation Results and Discussion section.

Once the energy in terms of Joule as well as in terms of amount of fuel is known, the results can be depicted. On one hand the results are the velocity of the car as a function of time, the corresponding displacement, energy consumption as well as cumulative energy consumption in terms of liter of diesel as a function of time. On the other hand the effects of the velocity and the traffic density on the fuel consumption on a shorter highway will be analyzed.

## 5.Simulation Results and Discussion

### 5.1 Use of the simulation model and general information

This code is an inspection tool as it returns the unit-cell velocity, which can be further used to determine other relevant quantities. We basically focus our project on inspecting the fuel consumption of two different cars on a highway varying the maximum allowed velocity as well as the traffic density.

Depending on user's interests, the code enables to evaluate the consumption, the unit-cell velocity, mean velocity, time of displacement, as well as the effect of a few parameters on those values.

In order to modify the code, the maximum allowed velocity on the highway can be handled in the highway_basecase function as well as the traffic density defined as the number of cars per unit length. Moreover the dawdling probability can be modified in order to take into account road conditions as well as driver behaviour.

The highway_basecase function is helped by a fuelconsumption function in order to evaluate the energy profile of the car. Few parameters can be modified here in order to match suitably the energetic yield. The parameters which can be varied are presented in table 2.

The computational power required to do the corresponding simulations was large, we therefore used computers with the following characteristics:

|  | Computer | Speed |
| :--- | :--- | :--- |
| Processor | Intel Core 2 Duo | 3.06 GHz |
| RAM | 4 GB | 1.07 GHZ |

Table 1: Computational power

Moreover, the useful characteristics of both cars in order to compute their respective energy consumption are as follows:

| Parameters | Large Car | Small Car |
| :--- | :--- | :--- |
| Length [m] | 1 | 1 |
| Cross sectional area $\left[\mathrm{m}^{2}\right]$ | 4 | 1 |
| Weigth [kg] | 2000 | 800 |
| Engine Efficiency [-] | 0.1 | 0.1 |
| Tyres friction [-] | 0.006 | 0.006 |
| Drag coefficient [-] | 0.3 | 0.3 |
| Air density [kg/m ${ }^{3}$ ] | 1.2 | 1.2 |
| Treshold engine energy [J] | 300 | 300 |

Table 2: Parameters of both kind of cars.

### 5.2 Mean velocity at different densities

First of all it is needed to know how the mean velocity depends on the density:


Figure 5: Mean velocity as a function of density at a highway length of 10 km . At a density of 0.01 the density decreases very quickly. Afterwards it becomes more and more constant. The blue and the black lines were done with the same parameters to see how the random car distribution changes the value of the mean velocity.

This measurement has been taken twice in order to observe the impact of the random car distribution over the highway. At a density of 0.01 the density decreases very quickly. Afterwards it becomes more and more constant. The general form of the mean velocity as a function of density suggest almost a step function.

The traffic jam needs to be defined and for that purpose, figure 5 is used: we stated that the highway reaches state of jam once the mean velocity of the car is almost constant. Therefore we have jam above a traffic density of 0.1 . This corresponds to ten cars of 1 meter length on a 100 m piece highway which seems to be relevant even in reality. The supposition of the small characteristic length of a car is here relevant as car length are almost always larger than space between them. Nevertheless it becomes questionable once the density reaches a certain value.

### 5.3 Displacement measurement over 100km and 1 km for the large car

For both cars the parameters were set in the following way:

| Simulation 1 | Large Car |
| :--- | :--- |
| Traffic density [-] | 0.001 |
| \# of cars on highway [-] | 100 |
| Dawdling probability [-] | 0.05 |
| Vmax [m/s] | 30 |
| Length of displacement [km] | 100 |

Table 3: Meaningful parameters for figure 6,7,9,10

Those properties lead to the following results for the large car:


Figure 6: Velocity, displacement, energy consumption and cumulative fuel consumption for a large car.

It shows on the upper left the velocity profile of our car as a function of time, we can notice some small fluctuations bounded in amplitude and coming from the dawdling probability. The upper right figure indicates that our car at a maximal speed of $108 \mathrm{~km} / \mathrm{h}$ reaches her place after around 58 min utes.

Below the velocity profile is the energy profile which will be used to compute for the cost of acceleration and the cost of inertia. Moreover, the friction loss due to the tyres needs to be compensated as well as the drag force accounting for the contact of the car with the air.

Finally, below the displacement as a function of time stands the cumulative fuel consumption as a function of time. After a short starting period, it starts to behave linearly with the displacement as the velocity fluctuations are not larger than $1 \mathrm{~m} / \mathrm{s}$.
The car is consuming about 21.5 I for 100 km , which stands for a car such as a Hummer H2.

The starting period, which holds almost 35 sec is depicted below:


Figure 7: The starting period of the large car. The first kilometre of figure 6.

As given by Nagel-Schreckenberg, the velocity is increasing linearly with the time up to the stationary value of $30 \mathrm{~m} / \mathrm{s}$. The displacement, given by the velocity multiplied by the time behaves as a quadratic function as both (velocity and time) do behave the same up to 35 sec . The energy is proportional to the velocity squared as it involves kinetic energy. Finally the fuel is also growing quadratically over this period.

Secondly, in order to investigate the effects of the maximal speed authorized as well as traffic density of the highway on the fuel consumption, the following experiments have been conducted:


Figure 8: Overall fuel consumption as a function of maximal speed for different traffic densities. Here the length is 1 km in order to save computational power. The other parameters are set equal to the other cases.

Those measurements have been taken over 1 km highway in order to save some computational power and as we know that the stationary states is reached before that length. On the upper left, we can see the fuel consumption of the car as a function of the stationary velocity. The fuel consumption is firstly decreasing down to a minimum and is then increasing. On the upper right, there are 10 cars on the 1 km highway. It seems to very slightly increase the latter.
Once we reached the state of traffic jam as in the last two figures, the consumption reaches a plate as the maximum velocity is defined by the jam. The stationary value becomes therefore a property of the traffic density and not anymore on the maximum velocity.

### 5.4 Small car

For the small car the same parameters as in Table $\mathbf{3}$ were used. With these parameters the simulation gives the following figures:





Figure 9: Velocity, displacement, energy consumption and cumulative fuel consumption for the small car.
The small car has a fuel consumption of 6.7 l on 100 km . Interesting is the very linear dependence of the fuel consumption as well as the displacement against the time. This comes from the more or less constant velocity of $30 \mathrm{~m} / \mathrm{s}$. That's also why the displacement is linear in the time. The zoom on the starting period in order to see the impact of acceleration in fuel consumption is showed in figure $\mathbf{1 0}$.

At this figure one is able to see directly what happens if the driver dawdles. At the middle of the velocity figure there can be seen, that the car does not accelerate over a cell. In this moment the change in the kinetic energy gets zero and the energy consumption gets reduced for a while. Furthermore we can see the same evolution of the energy as in figure 7. It seems that the kinetic energy has the greatest influence in this part.

Figure 11 seems very similar to figure 8. At low maximum velocities the fuel consumption gets extremely high due to the threshold energy term. After getting to the minimum, which is at about 31 for 100 km , the fuel consumption is linear to the maximum velocity. At higher densities at the end there is a constant fuel consumption because the car is in a traffic jam and does not get to the maximum velocity.


Figure 10: Velocity, displacement, energy consumption and cumulative fuel consumption of the small car at the first kilometre of figure 9.


Figure 11: fuel consumption as a function the maximum velocity at different densities and at a length of 1 km

### 5.4 Discussion over the difference between both cars

While recording the displacement measurements, the traffic density was of 0.001 . The mean velocity is exactly the same in both cases which is accounting for the fact that the dawdling probability of 0.05 is too low to have any impact on mean velocity over such a large distance.

|  | Small car | Large car |
| :--- | :---: | :---: |
| mean velocity $[\mathrm{m} / \mathrm{s}]$ | 29.8 | 29.8 |
| fuel consumption $[\mathrm{L}]$ | 6.7 | 21.5 |

Table 4: 100 km displacement. These values refer to the simulations at figures 6,9.

If we would like to observe the effects of the dawdling probability on the mean velocity, we should record to velocity on a length smaller than 20 m or run a sensitivity analysis over the dawdling.

The sensitivity analysis over the maximum velocity while recording fuel consumption enables us to compare the effect of the car size on the fuel consumption.

The threshold fuel consumption was chosen to be the same for both cars. It has therefore a larger effect on the small car than on the large one. Conclusion is that we should drive faster with a smaller car than with a larger one to obtain the highest yield of fuel with respect to a given distance.

The differences between the maximum yield of the small car at no traffic density and in a jam is roughly 2 L for a 100 km displacement as it is of 4.5 L in case of large car. It is therefore better as intuitively expected to undergo jam with a small car, even if the threshold consumption is the same. Actually the threshold consumption will depend on the size of the engine and therefore won't be the same for both, it is then an assumption and the model could be improved that way.

We can observe that the range of the consumption is very different for both cars at least at low density where the velocity range is not reduced by jam. It goes from 3 to 11.5 L for the small car as for the large one it evolves between 5 and 23L. This range difference is mainly explained by the air friction which strongly depends on the cross sectional area of the car.

The simulation of the fuel consumption as a function of the maximum velocity was just done over a length of 1 km . This distance is just above the starting period. We can argue according to figure 11 that at the end there is a boundary effect for low densities due to the impossibility of cars to keep the stationary velocity as they start to feel the end of the highway. In order to avoid this undesired effect, those kinds of sensitivity should be run either on a larger distance or for a lower maximum velocity. This is explaining why it seems to be a plane for maximum velocities roughly above $35 \mathrm{~m} / \mathrm{s}$. According to reality, the fuel consumption should even grow faster than linearly with a constant increase of speed.

Nevertheless the length of one 1 km is still relevant for the other values. By scaling up the fuel consumption over 1 km with the one in table 4 at a maximum velocity of $30 \mathrm{~m} / \mathrm{s}$, we get similar values.

Furthermore with respect to the previous investigations, traffic jam is not occurring until a car density of 0.1 . Then the maximum allowed velocity is no longer reached and so the energy consumption reaches a plane.

As expected, by increasing the traffic density the fuel consumption increases independently of the car properties. Table 5 shows the evolution of the consumption at different densities for a maximum allowed velocity of $30 \mathrm{~m} / \mathrm{s}$. This corresponds almost to the upward limitation in Switzerland which is of $120 \mathrm{~km} / \mathrm{h}$. The first row accounts for the fact that the large car will consume more than the small one, second row can be interpreted as if there are 10 cars on a 100 km length highway then there is still no other car felt.

| Density | Small car consumption [L/100km] | Large car consumption [L/100km] |
| :--- | :---: | :---: |
| 0 | 7 | 20 |
| 0.01 | 7 | 20 |
| 0.1 | 5.5 | 10.5 |
| 0.2 | 8.5 | 13 |

Table 5: Fuel consumption as a function of traffic density.

This can be explained with help of figure 5. There we see that the mean velocity is almost constant at this density and close to the maximum one. Therefore the fuel consumption is also constant.

The third row shows us that in a jam the difference between the consumption of both cars is reduced as they are limited in speed, the small car is still better. Finally the last row shows us that as we increase the traffic density, the consumption is increased. Furthermore the effect of increasing the density is the same for both cars which is relevant according to the model.

## 6.Summary and Outlook

The model presented describes the fuel consumption of a car in a relevant way, as values seem realistic. So the introductory questions can be answered:

- Fuel consumption depends as well on the maximum velocity. If cars got stuck in a jam (traffic density over 0.1 ) it never gets to the maximum velocity and suddenly the fuel consumption decreases. On the other hand, as the mean velocity is also decreased, time of transport is extended. Therefore we face a drawback which should be further explored in order to find the best solution.
- In table 5 there can be seen that the fuel consumption depends more on the car size than it depends on traffic density.

First of all, according the latter comments, the most important choice is to avoid traffic jam in order to decrease the fuel consumption. A solution could be to transport shipment and people during night time, where the density is lower.

To decrease the consumption further, we should try to avoid using larger vehicle than really needed.
Finally and in order to improve this model the following ways are proposed:

- Involve a second lane on the highway.
- Acceleration was assumed to be instantaneous, which is obviously not the case in real life. Improving this drawback can be done for instance by calculating the acceleration as a function of the distance between the car in front of us.
- The assumption of the small characteristic length of car is relevant as long as the traffic density is not too high.


## 7.References

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