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Simulating artificial financial markets

## Simulating the stock market including the formation of bubbles and crashes of the stock market

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## Abstract

The objective of the simulation :

The objective of this project is to simulate in a realistic way the stock market including one of the most interesting and important phenomenon in the bubbles and crashes. The bubbles are spectacular rises in the prices of the shares in a short time . Often subsequent to bubbles are crashes the spectacular fall in prices of shares in a short time . In social sciences , such spectacular rises and subsequent falls can be seen in many circumstances , like the rise and fall of empires , the rise and fall of value , the rise and fall of civilizations , the rise and fall of political man . The phenomenon is however similar to one occurring in physics , the boiling of water or other substances . Considering now the objectives of the simulation , my first objective is a very clear presentation of the phenomenon , more detailed than in the scientific articles of quantitative finance, which assume knowledge about problem domain , the stock market. My second objective is to develop and present a system to simulate the dynamics of the stock market , based on trading agents or control variables as they are known in system theory.

The model implemented :

I will use concepts from artificial intelligence , like multi-agent societies and the social interaction of agents from these societies . Also I will use concepts from control theory and systems like control parameters , estimation , observable parameters .
I will simulate a market with traders who base their decisions on the economical fundamentals of the corporations who's shares are traded and also traders who base their decision on the change in prices of the shares .

```
Expected results :
The simulation has the aim of presenting in a realistic way
the dynamics of prices , the causes of price formation and
the typical stages in a bubble and the typical stages in a
crash as they can be observed in historical prices of the
shares .
```

I will start with a introduction to the domain of simulation .
Relevance of the problem :
On the study of stock market interaction and how fundamental is to the understanding of the social life and social sciences and exact sciences

The oldest type of markets that resembled the stock market were early wheat and corn markets in the Roman Empire
The stock market is like any trading markets were exchanges take place. However looking to life in general we see trading occurs in many cases of human interaction. We can see trading in every decision taken in the interaction of people. In trading sometimes goods change hands or goods for money or sometimes illusion for money or real innovation for money or labor to money or pleasure for money and the list can go on . Anything that is a fundamental need and it is rare resource ( any thing that is not free in nature ) can be the object of trading. The universal mediator is the money, a tool that intermediates, the exchange of goods using their approximate tradeoff value in money. It is like a game when you trade pieces in an advantageous way, it could be fun and it could be risky, it could bring position and wealth and could get you life experience like no other thing can do .

## 1. The motivation

The stock market is one of the most fascinating games, a place were logical decision as well as luck could result in large fortunes but also in ruin .

Many call it the big casino because the amount of the transactions exceeds that of any casino. There are many stock exchanges, each having its own rules and the behavior of prices it's own statistical properties. The biggest stock exchange is the famous New York stock exchange, on Wall Street, New York City .


This place has seen some of the most dramatic events of the late $19^{\text {th }}$ and of $20^{\text {th }}$ century, including the rise of United States from an average economy to the biggest economic power. This event has taken part in the late part of the $19^{\text {th }}$ century and first part of the $20^{\text {th }}$ century , 1880-1920. This resulted in the exponential growth in the value of the assets valued on New York Stock exchange.

One of the most dramatic events of the $20^{\text {th }}$ century has been the great crash of the stock exchange in 1929.


One can see the bubble ( the big increase in prices ) between 1923 and 1929 and then the crash (decrease in prices ) of 1929 , the great crash .

This event is considered the start of the Great Depression, with complex consequences on the society and in the rise to power of the main actors of the second world war, the greatest military conflict of the history .

The most important events in history had some connection with the stock exchange.

The stock exchange has seen the rise and fall of many speculators and traders , famous banks and famous bankers, and the drama of many ordinary people .

Some of the central events including far reaching catastrophes have taken the form of bubbles and crashes, the phenomenon I study in this paper .

## 2. The phenomenon studied

The stock market is a general case of market where people and companies can buy and sell physical objects like commodities or imaginary objects, like shares in companies, shares called stocks .

The stock exchange functions upon the laws of offer and demand. A transaction will occur where the offer meets the demand. The mechanisms have been changed very much in time . Probably the first modern stock exchange is the Amsterdam stock exchange. The biggest stock exchange is the New York Stock exchange. The beginning of New York Stock Exchange has been in an auction system and of course the stock can be seen as an auction system. The auction was held in the early $19^{\text {th }}$ century under a tree on the street called Wall Street in the place that is know the famous street of banks, Wall Street, New York City, New York .


The famous skyline if Manhattan the financial center of New York City :


The market bubbles are fast increases in the price of the stocks over relatively short time. The crashes are fast decreases in price over a short time span .

It is not necessary that a bubble to be followed by a crash, sometimes is followed by a plateau .

Also, it is not always that a crash was the result of a bubble but often it is like this .

Let us analyze the phenomenon by studying several cases .
For example the dot net bubble, the increase in prices of the information technology stocks at the end of 90' and start of 2000.


During the dot net bubble the expectation about the new economical paradigm, based on information technology was so high that prices of the shares were not correlated at all with the fundamentals of the companies . After a justified optimism in the new it sector , between 1994 and 1999 there is a huge increase followed by a fast decline in prices .

Bubbles can also take happen to broad indices like dow jones. For example look at the bubble before the 1929 crash .


It can be seen a bubble then the sharp decrease in prices. The initial increase in prices from 1900 to 1920 is the result of the growth of the Unites States economy and has fundamental causes, the exponential increase at ever higher speed after that is the bubble and it was not any more correlated with underlying fundamentals. The correction took place in the form of the crash .

In the following picture you can see the panic of investors in the day of the great crash :


Bubbles can be formed also in real estate .
For example the real estate bubble in Japan


During the real estate bubble in Japan, the prices of commercial real estate increased very much. During this time, the price of the area of the Imperial Palace in Tokyo increased up to the level when it had a value equal to the land in the entire state of California and also to the entire land in Canada. Of course the prices could not be sustained and decreased at a sharp rate .
The first stage has been when the prices started to increase as a logical consequence of the increase in industrial activity in Japan. The next stage however has been a speculative phase when many speculative investors entered in the game and bought a lot of real estate. The banks were lending money very easy and there was plenty of money to buy on credit.
At some point there was not enough money on the market to fuel the exponential growth of the prices and the bubble transformed in a crash.

A similar situation happened with the real estate bubble in the USA that generated the greatest economic crisis since The Great Depression:


As it is known from the news, the sequence of events was :
First the housing prices started to increase because the growth in the IT sector and in other parts of the united states economy. Then when the money on the market was not enough anymore the building of new houses was made on credit and then new instruments like derivatives emerged as a tool of debt management. Eventually once some of the banks started to fall then the entire system has been put at risk and the united states government bailed out the biggest institutions .

In the next chart can be seen the crash of the stock exchange in 2008 :


The causes that determine the bubbles and crashes are largely unknown and are an open problem .

Considering however the fast variation in prices, that happened with sufficient frequency to suggest the idea the variation in prices do not follow a Gaussian distribution, but a different one.

The problem is scientifically challenging because it is very complex and it is also interesting. Analyzing such problems could be a way to understand one of the sources of big money, the financial speculation in the stock exchange .

## 3. The methodology

I formulate several statistical hypothesis and then try to test them experimentally by simulation.
The objective of the simulation is to test the following hypothesis :
H1. The number of big large variations may be because the crowd effect , were agents imitate each others behavior .

H2. When the number of rational traders is small, bubbles often occur . This may be observed by the fact that the market price of the companies is bigger than the price resulted from fundamental analysis .

H3. When the number of rational traders is large , the market price is generally locked within the price range consistent to fundamental analysis .

H4. The short term fluctuation in price are endogenous to the dynamics of the stock market but on the longer term the fundamentals prevail .

H5. The big variations of prices are to frequent so their distribution cannot be described by a Gaussian distribution. Maybe it could be modeled by other distribution, for example the Levy distribution.The main problem in describing the prices with a Gauss distribution is that large events in the Gauss distribution are unlikely.

## 4. The analyzes of the system

I define here rational traders as traders who take investment decisions based on the fundamental value of the stock .

I define here a 'noise trader' as a trader whose decisions are taken by studying the market dynamics, the changes in the prices. The decisions of noise traders are more likely to imitate the decisions of others .

Mathematically, the system is a Stochastic Dynamic System . Practically , the system is a complex social system one of the most unique type evolution has seen.

Only humans display such advanced type of social behavior, capable of building such complex interaction. To isolate the essence we are interested only in the dynamic of the price.

Because the price reflects the value of a share in corporations, so shares in real economy , the price of the shares is influenced by economic changes at microeconomic and macroeconomic level as well as by the international economic system .

If the effects like bubbles and crashes can be simulated using internal parameters of the system than this will suggest an endogenous cause for bubbles and crashes .

## 5. The elements of the system

A. The most important element to be taken in account is the trading agents , fundamental traders and noise traders .
B. The transactions based on supply and demand principle .
C. The relation between the price and the balance between offer and demand .
D. The relation of the offer with the price.
E. The relation between the economic fundamentals and the price .
F. The relation between the general economy and the expectation of the public.
G. The relation between the general economy and the performance of the stock.
H. The existence ( assumption ) of a positive feedback that determines the investors to invest in rising stocks .

## 6. Approximations and Limitations

Every model in theoretical sciences and in experimental sciences has assumptions and approximations. The better the assumptions and the closer to reality, the better the model.

The model is realistic but it is necessary to make some assumptions :
(1) approximation : The price formation mechanisms . I assume the orders are executed instantly a thing not always true . I do not consider the margin call pressure from the brokerage companies and banks and other constrains in the decision of the investors.

## Reasons for the approximation :

It is impossible to know all the details of the system. If all the details about the structure and processes of the system would be known, no approximation would be necessary .
limitation : There is no general theory that will describe the evolution of prices in the stock exchange and there are not known all the details that contribute to the formation of prices .
(2) approximation : We must consider a number of initial conditions for the system. The variables may be considered as dependent or independent. The controllable variables are independent from the system functions. The dependent variables are dependent on the processes of the system .

Reasons for the approximation :It is impossible to know all the values of the system parameters at a given time.
limitation: We cannot know what is in the mind of investors. The price of the stock may be influenced indirectly by many factors, but ultimately the decision of investors to buy and sell a certain stock will count .
(3) approximation : In the real stock, measuring the parameters could change their values but in my model as in many such research models, measuring the
parameters will not change the values of these parameters. ( at least for the beginning )

## Reasons for the approximation :

Measuring means trading but trading will change the prices in some degree .
limitation : Some parameters may be known only by direct investment and this action will change the prices .
(4) approximation : Generally, I will assume the price of the stock is influenced or correlated with the following external factors :
A. the value of the Dow Jones industrial average
B. the state of the world economy
C. the Dow Jones is also function of the state of the economy where the stock market operates
D. the state of the economy of the country or countries where the company operates
E. the state of the world economy

## Reasons for the approximation :

It is impossible to know al the interactions of the system with other external systems .

## limitation :

The stock is a complex system that interacts with many external systems : the economic system of the country where it operates, the international economic system, the financial markets from different countries, the politics of governments and many other factors .
It is not fully understood how these elements interact and influence the stock exchange. Even if we could know something about how isolated events external to the stock system influence the system it is much harder to understand how these factors interact in a combined action .
(5) approximation : The purpose of the simulation is to verify some hypothesis about the causality of some stock exchange phenomenon.

## Reasons for the approximation :

There is no theory to explain the causes and effects in a precise way .
limitation : There are many theories that explain in a qualitative way the factors influencing the dynamics of the stock exchange, however few theories will explain the complex interaction with a general theory that will integrate in the same framework all the forces acting on the market prices in the stock exchange .
(6) approximation : The model must be in general computable and my model is .

## Reasons for the approximation :

The idea of Pierre Simon de Laplace was that if there would be known all the details of the world and there would be vast intelligence, then the future will be clear as the past for such intelligent entity. So , even at that time they were realized two important limitations : the perfect knowledge and the computation power. Even if we could know and store all the data about a system, it may be a very difficult computational task to "compute the future"
limitation : Even if we could get all the data we wanted and even if a general theory would exist, the computational power required will be great. In the bound of the existing, very limited and approximate models it is still a very complex task to program a stock trading system to take decisions in real time.
(7) approximation : The future dynamics of this artificial stock will not be changed by the simulation, because in reality, the emergence of a predictor of the stock market would change the future dynamics because information about the predictions will spread and determine the behavior of the investors .

## Reasons for the approximation :

It may be the most important limitation in some systems : predicting and simulating the future may change it .
limitation : Even if all the condition would be right, something impossible as long as we do not have a precise theory of how people take decisions, and even if we have a system that could predict the future prices given the past prices, the information would leak out of the system and there will be people trying to take advantage and they will change the future dynamics of prices .

## 7. System implementation

## Modeling investors

The system starts with a number of trading agents, given as initial condition of the system. At each step in the simulation a trader will update his bid by a unit , buy or sell . The decision is taken according to the investment style of that trader but also using a randomly generated value to introduce randomness in the system. The number of agents taking part in the simulation will be increased until the statistical properties will be approximated .

- modeling the output
the output is the files with data that can be plot. At each step or at a number of steps the program shall output a file with data .


## 8. Control strategy

The specifications would be :

- to stop the system on certain conditions : number of seps
- the system should output data at certain discrete time frames . ex: 10xdt
- the output should be in a way that can be visualized
- the system should be programmed to work during certain time frames :
ex: $y * d t->x * d t$
- the system should be able to recognize the main stages of the simulation


## 9. The modeling of the system

The mathematical model of the system:
The model of the stock exchange in this simulation is a dynamic stochastic system.
System analyzes:
The estimated quantity : the price
The feedback quantity : the price

The control quantities :

- the state of the economy
- the fundamentals of the company
- the number of investors
- the number of fundamental investors
- the number of noise traders
- the initial price

The price formation mechanism :

- the combined effect of bids at one time step will influence the price directly

The observable quantities :

- the price
- the number of buy orders
- the number of sell orders

The observable correlations :

- stock price / fundamentals
- stock price / number of fundamental investors
- number of fundamental investors / number of noise traders


## 10. The structure of the program

First the control is taken by the main function. From here a menu is displayed and the control passes to the option function. The option function interacts with the user and receives the input then passes the control to the main function which in turn gives the control to the simulate function that performs the process and then returns the data and control to the main function. If the user chooses readme the information is displayed in console .
The main processing is done in the simulate function. The simulation takes place by iterating through the time steps of the discrete time system using a for instruction . The function "main" calls the function "option" do display a menu consisting of the following options:

```
1. readme' ,
2. '2. simulate',
3. '3. display the price vs time graph' ,
4. '4. display the price vs fundamentals graph' ,
5. '5. display the number of fundamental investors VS number of
    momentum investors at this time step' ,
6. '6. display the number of fundamental investors vs price' ,
```

7. '7. display the number of momentum investors vs price',
8. '8.display the number of fundamental investors vs fundamentals', 9. '9. display the number of momentuum investors vs fundamentals ', 10. 10. momentuum investors who invest at each time step vs time' ,
9. '11. display the number of fundamental investors VS number of momentum investors who have invested in the stock until now' ,
10. '12. display fundamentals vs time' ,
11. '13. time VS total number of fundamental investors investing after each time step' ,
12. '14. display the total number of momentuum investors vs time' ,
13. '15. display the total number of fundamental investors vs price' ,
14. '16. display the total number of fundamental investors who invest vs time ' ,
15. '17. display the number of momentuum investors who invest at each step vs fundamentals' ,
16. '18. exit'

## 11. The code and description of its functions

## Function readme :

The function readme offers information about the functions and objectives of the simulation and about the functions of the control panel .

```
function readme
%this function explain a little bit about e \
```



```
fprintf(' The purpose of the program is to simulate in a realistic way
processes in a stocastic dynamic system ,\n ')
fprintf(' the stock market as well as the general dynamics of the
market\n ')
fprintf(' Using some possible imput paramters the program outputs data
as well as graphs\n ')
fprintf(' -------------------------------------------------------------------------------
fprintf(' -----------------------------------------------------------------------------
fprintf(' InputData data :\n ')
fprintf(' ---------------------------------------------------------------------------------
fprintf(' number of fundamental investors ,\n ')
fprintf(' number of momentum investors ,\n ')
fprintf(' the fundamental value of the company at the beginning \n ')
frpintf(' the change in fundamentals during the time of the simulation
\n')
```

```
fptintf(' the starting price\n ')
fprintf(' ------------------------------------------------------------------------
fprintf(' ---------------------------------------------------------------------
fprintf(' the terms used :\n ')
fprintf(' ------------------------------------------------------------------------
fprintf(' fundamental investor means here investor who bases his
decisions on the fundamentals of the company\n ')
fprintf(' momentuum investor means here investor who bases his
decisions on the variation in the prices of stocks\n ')
fprintf(' the fundamentals of a company mean here economic performance
of the company \n')
fprintf(' for modeling and abstraction purposes I agrgated the economic
performance in a single variable\n ' )
fprintf(' economical data about the company can be obtained from the
three main financial statements the companies have to present\n ')
fprintf(' at the end of each quarter :\n ')
fprintf(' the income statement , cash flow , balance statemen\n ')
fprintf(' these papers contain economic data describing the performance
of a company such as :\n ')
fprintf(' Total Revenue , Gross Profit , Total Operating Expense ,
Income Before Tax , Income After Tax , Net Income Before Extra. Items
\n')
fprintf(' Net Income , Income Available to Common Excl. Extra Items ,
Income Available to Common Incl. Extra Items ,')
fprintf(' Basic EPS Including Extraordinary Items , Diluted EPS
Including Extraordinary Items , Normalized Income Before Taxes ')
fprintf(' Normalized Income Avail to Common , Total Current Assets ,
Total Assets , Total Current Liabilities , Total Long Term Debt ')
fprintf(' Total Debt , Total Liabilities , Total Equity , Total
Liabilities & Shareholders Equity , Total Common Shares Outstanding ')
fprintf(' Cash from Operating Activities , Cash from Investing
Activities , Cash from Financing Activities , Net Change in Cash ')
fprintf(' while all these financial data cannot always be aproximated
in a single varaible because the economic and financial position of ')
fprintf(' a company has more components however we may use a single
value in order to aproximate the relative strength of busines
performance')
fprintf(' such an idea of making the evaluation of a position is used
also in coputer chess , where the evaluation function returns the
aproximate ')
fprintf(' value of a chess position . In this case the evaluation
function will return the value of the buneinss pasition of a
corporation . ')
fprintf(' In this program the variable I called "fundamental value" has
such a function. The investors who base their decision on such a value
')
fprintf('I called here "fundamental investors" . The term "fundamental
investor" is also widelly used in business')
fprintf(' ----------------------------------------------------------------------------
fprintf(' ---------------------------------------------------------------------------------
fprintf(' names and content of input files : ')
fprintf(' ------------------------------------------------------------------------------
fprintf(' input.dat ')
fprintf(' the data is on the first line of the file \n')
fprintf(' position 1 : number of fundamental investors ,\n ')
```

```
fprintf(' position 2 : number of momentum investors ,\n ')
frpintf(' positions 3-13 : the change in fundamental economic
performance of the company during the time of the simulation \n'
fptintf(' position 14 : the starting price\n ')
fprintf(' position 15 : the fundamental value of the company at the
beginning \n ')
fprintf(' ------------------------------------------------------------------------------
fprintf(' --------------------------------------------------------------------------------------
fprintf(' Exit data :\n')
fprintf(' ---------------------------------------------------------------------------------
fprintf(' the plots in the generated by selecting choices in the menu
')
```


## function main :

```
% Call a function to display a menu get a choice
```

```
choice = option;
% choice 18 is to exit the program
while choice ~=18
    switch choice
        case 1
            % describe the simulation and the output files
            readme;
        case 2
            % print the input data
            % limite;
```

[fundamentalsVec, divisionNumberVec, priceVec,timeVec,extendedFundamental sVect, fundamentalInv, momentuumInv, numberOfFundamentalInvestors, numberOf
MomentuumInvestors] = simulate
case 3
\% display the price vs time graph
plot(timeVec,priceVec)
title(' ox - time | oy - price ')
case 4
\% display the price vs fundamentals graph
plot(timeVec,extendedFundamentalsVect,'r')
hold on
plot(timeVec,priceVec,'b')
title(' price vs time - blue | fundamentals vs time - red
')
\% display the number of fundamental investors vs momentuum
investors at each time step vs time
plot(timeVec, momentuumInv,'r')
hold on
plot (timeVec, fundamentalInv, 'b')
title('time vs number of fundamental investors investing at
each step (b) vs momentuum investors (r) ')
case 6
\% display the number of fundamental investors vs price
plot(timeVec, fundamentalInv,'r')
hold on
plot(timeVec, priceVec, 'b')
title(' price vs time - blue fundamental investors who
invest at each time step vs time - red ')
case 7
\% display the number of momentum investors who invest at
eact step vs price
plot(timeVec, momentuumInv,'r')
hold on
plot (timeVec, priceVec, 'b')
title(' price vs time - blue | momentuum investors who
invest at each time step vs time - red ')

```
    case 8
    % display the number of fundamental investors who invest at
each step vs fundamentals
    plot(timeVec,fundamentalInv,'r')
    hold on
    plot (timeVec, extendedFundamentalsVect,'b')
    title(' fundamentals vs time - blue | fundamental investors
who invest at each time step vs time - red ')
```

```
    case 9
        % display the number of momentuum investors who invest at
each step vs fundamentals
        plot(timeVec,momentuumInv,'r')
        hold on
        plot(timeVec,extendedFundamentalsVect,'b')
        title(' fundamentals vs time - blue | momentuum investors
who invest at each time step vs time - red ')
    case 10
        % display the number of momentuum investors who invest at
each time vs time
            plot(timeVec,momentuumInv)
            title(' momentuum investors who invest at each time step
vs time ')
    case 11
    %the total number of fundamental investors who deposited
money in
    %that stock vs the total number of momentul investors who
deposited
        %money in that stock
        plot(timeVec, numberOfFundamentalInvestors,'b')
        hold on
        plot(timeVec, numberOfMomentuumInvestors,'r')
        title('the total number of fundamental investors(blue) vS
the total number of momentul investors(red) ')
        case 12
        % display the fundamentals vs time graph
```

```
    bar(divisionNumberVec, fundamentalsVec)
    case 13
        % display the total number of fundamental investors at each
time step vs time
        plot(timeVec,numberOfFundamentalInvestors)
        title('time VS total number of fundamental investors
investing after each time step ')
    case 14
        %display the total number of momentuum investors vs time
        plot(timeVec, numberOfMomentuumInvestors)
```

```
    title('time VS total number of momentuum investors
investing after each time step ')
    case 15
        %display the total number of fundamental investors vs price
        plot(timeVec,priceVec,'r')
        hold on
        plot(timeVec, numberOfFundamentalInvestors,'b')
        title(' fundamental investors vs time - blue | price - red
')
    case 16
    % display the total number of fundamental investors who
invest vs fundamentals
        plot(timeVec, numberOfFundamentalInvestors,'r')
        title(' fundamental investors who invested until now vs
time - red ')
        case 17
        % display the number of momentuum investors who invest at
each step vs fundamentals
        plot(timeVec,extendedFundamentalsVect,'r')
        hold on
        plot(timeVec,priceVec,'r')
        hold on
        plot(timeVec, extendedFundamentalsVect-priceVec,'b')
        title(' fundamentals - price vs time - blue | fundamentals
- price - red ')
```

```
    end
    %display menu
    choice = option;
end
```


## the simulate function :

```
function
[fundamentalsVec,divisionNumberVec,priceVec,timeVec,extendedFundamental
sVect,fundamentalInv,momentuumInv, numberOfFundamentalInvestors, numberOf
MomentuumInvestors] = simulate
%--------------------------------------------------------------------------------
----
% role of the function : simulates the market
%-
----
%imput data :
%read the number of fundamental and momentuum investors read the number
of time steps
%possitions in the entry file and then entry vector :
%position 1. number of fundamental investors ;
%position 2. number of momentuum investors ;
%position 3. number of time spteps of the system simulation ;
%position 4 - 13 : change in fundamental performance of the company
%the 14th position in the vector represents the starting price
%the 15th possition in the vector represents the starting value of the
%fundamanental performance of the company
%----------------------------------------------------------------------------------
----
%output data :
% fundamentalsVec - the vector containing the data with the performance
of the company , read from the file
% divisionNumberVec - contains the moments when the fundamentals are
% published
% priceVec - contains the price resulted from simulation at each step
% timeVec - the discrete moment of time that are the steps in the
simulation
% extendedFundamentalsVect - because the fundamentals change slower
that a
% trading steps in order to plot the variatio of fundamentals and the
variation of
% other variables which change valeu at each time step it is needed to
fill
% a vector with the same size as the time vector and where there is a
value
% of the fundamentals at each time step, but of course changes in
value
% will apear only when changes in fundamentals occur
% fundamentalInv - the vector contains the number of fundamental
investors
% who invested at each simulation step
% momentuumInv - the vector contains the number of momentuum investors
who
% invested at each simulation step
```

```
% numberOfFundamentalInvestors - contains the total number of
fundamental
% investors buy action after one step - this means the summation of all
the
% buys and substraction of all sells at previous steps
% numberOfMomentuumInvestors - contains the total number of momentuum
% investors buy action after one step - this means the summation of all
the
% buys and substraction sells at previous steps
%---------------------------------------------------------------------------------
---
% read data from files
fid = fopen('input.dat')
%if the file was not found
if fid == -1
    disp('File open not successful')
else
            oneline = fgetl(fid)
            %separate each line into the number and character
            inputData = str2num(oneline)
end
closeresult = fclose(fid);
if closeresult == 0
    disp('File close successful')
else
    disp('File close not successful')
end
% fundamentalInv - number of fundamental investors
% momenumInv - number of momentum investors
%initialize the vectors coresponding to the agents
% v1 contains the type of the agent 1 for fundamental 2 for momentuum
% v2 contains the number of trades the agent performed
% v3 contains the value the agent won or lost
for i=1:inputData(1)
    v1(i) = 1;
end
for i=1:inputData(1)
    v2(i) = 2;
end
%processing part
```

```
%print the data to output
    fprintf('number of fundamental investors : %.2f\n ', inputData(1) )
    fprintf('number of momentuum investors : %.2f\n', inputData(2) )
    fprintf('number of time steps of the simulation
%.2f\n',inputData(3) )
    disp(' the fundamental performance of the company:' )
    %print the values of the fundamental behaviour of the company
    for i=4:13
            fprintf(' frame %d : %.2f\n ' , i-3 , inputData(i) )
    end
    fprintf('the starting price of the company stock
%.2f\n',inputData(14) )
    fprintf('the starting fundamental value of the company
%.2f\n',inputData(15) )
    %perform the simulation
    %initialization
r = rand(1)
offer = 0
demand = 0
currentPrice = inputData(14)
currentFundamentalValue = inputData(15)
wealthIn = 0
division = inputData(3)/10.0
currentDivision = 0
divisionNumber = 1
index = 1;
%the initilization step
    fundamentalsVec(index) = currentFundamentalValue
    divisionNumberVec(index) = divisionNumber
    currentDivision = division + currentDivision
    divisionNumber = divisionNumber + 1
    index = index + 1
    currentFundamentalValue = currentFundamentalValue + inputData(2
+ divisionNumber )
```

    agents1 = inputData(1)
    agents2 \(=\) inputData(2)
    ```
price = inputData(14);
    fundamentalInv(1) = 0;
momentuumInv(1) = 0;
    numberOfFundamentalInvestors(1) = 0
    numberOfMomentuumInvestors(1) = 0
    for t=1:inputData(3)
% process all the time steps
%simulation strategy :
    %the loop of the time step - I assume at eac time step all the
traders
    %will decide in buying or selling or doing nothing , but will still
have
    %to decide among the three options
%the loop going through the list of traders is assesing if they decide
to
%buy or sell - if the price they evaluate the stock is bigger than the
%assesed price buy with a probability else, if the price they evaluate
the stock is lower than the
%assesed price , sell with a probability
    fundamentalInv(t) = 0;
    momentuumInv(t) = 0;
    %go through the fundamental agents
    for a=1:agents1
        if v1(a) == 1
            if currentPrice < currentFundamentalValue
                r = rand(1)
                if abs(r + 0.2) > 0.5
                    demand = demand + 1;
                            fundamentalInv(t) = fundamentalInv(t) + 1;
                    % if wealthIn < 50000
                    price = price + ( 2.0)/(inputData(1) +
inputData(2));
                        wealthIn = wealthIn + price
                    % end
                            currentPrice = price;
        else
                            demand = demand - 1;
                    if t > 1
                        % if numberOfFundamentalInvestors(t-1) > 1
                                fundamentalInv(t) = fundamentalInv(t) - 1;
            % if wealthIn > 5000
            % if price > inputData(1)/10
                                price = price -
( 2.0 )/(inputData(1) + inputData(2));
```

```
                                    wealthIn = wealthIn - 0.5*price
                                    end
                            % end
                            %end
            end
                        currentPrice = price;
            end
                                    elseif currentPrice >= currentFundamentalValue
                                r = rand(1)
        if abs(r + 0.2 ) > 0.5
        demand = demand + 1;
        if t >1
        % if numberOfFundamentalInvestors(t-1) > 1
                        fundamentalInv(t) = fundamentalInv(t) - 1;
                % if wealthIn > 5000
                        if price > inputData(1)/10
                price = price -
( 2.0)/(inputData(1) + inputData(2));
                                wealthIn = wealthIn - 0.5*price
                        end
                        % end
            end
                        % end
                    currentPrice = price;
                        else
        demand = demand - 1;
        fundamentalInv(t) = fundamentalInv(t) + 1;
            % if wealthIn < 50000
                        price = price + ( 2.0)/(inputData(1) +
inputData(2));
                                wealthIn = wealthIn + price
                        % end
                        currentPrice = price;
            end
            end
        if t==1
            numberOfFundamentalInvestors(t) = 0;
        else
            numberOfFundamentalInvestors(t) =
numberOfFundamentalInvestors(t-1) + fundamentalInv(t);
        end
    end
extendedFundamentalsVect(t) = currentFundamentalValue;
priceVec(t) = price;
```

```
    timeVec(t) = t;
    if t > currentDivision
        fundamentalsVec(index) = currentFundamentalValue
        divisionNumberVec(index) = divisionNumber
        index = index + 1
        currentDivision = division + currentDivision
        divisionNumber = divisionNumber + 1
        currentFundamentalValue = currentFundamentalValue + inputData(2
+ divisionNumber )
    end
    end
    %go through the momentuum agents
    for a=1:agents2
        %select a number randomly from previous 10 time steps
        if t > 1
            if t < 10
            rx = 1
            else
                rx = randint(1,1,[1,10]);
            end
            indx2 = t-rx+1
            indx1 = t-1
            if v2(a) == 2
                if priceVec(indx2) < priceVec(indx1)
                        r1 = rand(1)
                        if abs(r1 + 0.2) > 0.5
                        demand = demand + 1;
                        momentuumInv(t) = momentuumInv(t) + 1;
                        if wealthIn < 50000
                                price = price + ( 2.0)/(inputData(1) +
inputData(2));
                                wealthIn = wealthIn + price
```

```
        %
                        end
                        currentPrice = price;
            else
                                demand = demand - 1;
                                if t > 1
                            % if numberOfMomentuumInvestors(t-1) > 1
                                    momentuumInv(t) = momentuumInv(t) - 1;
    %
inputData(2));
    %
                                    end
                                currentPrice = price;
                        end
            end
            end
        elseif priceVec(indx2) >= priceVec(indx1)
            r1 = rand(1)
            if abs(r1 + 0.2 ) > 0.5
                demand = demand + 1;
                if t > 1
                        % if numberOfMomentuumInvestors(t-1) > 0
                        momentuumInv(t) = momentuumInv(t) - 1;
                        %if wealthIn > 5000
                                    if price > inputData(1)/10
                                    price = price -
( 2.0)/(inputData(1) + inputData(2));
                            wealthIn = wealthIn - 0.5*price
                        % end
    %
                        end
        % end
        end
                currentPrice = price;
        else
        demand = demand - 1;
        momentuumInv(t) = momentuumInv(t) + 1;
    %
            if wealthIn < 50000
                        price = price + ( 2.0)/(inputData(1) +
inputData(2));
                        wealthIn = wealthIn + price
            % end
                currentPrice = price;
            end
            end
        end
end
```

```
    if t==1
        numberOfMomentuumInvestors(t) = 0;
    else
    numberOfMomentuumInvestors(t) =
numberOfMomentuumInvestors(t-1) + momentuumInv(t);
    end
    priceVec(t) = price;
```

end
end
end

## The function option :

function choice $=$ option
\%print the menu of options and error check
\%until the user pushes one of the buttons
choice $=$ menu(' Select an options', '1. readme', '2. simulate', '3. display the price vs time graph' , '4. display the price vs fundamentals graph' , '5. display the number of fundamental investors VS number of momentum investors at this time step', '6. display the number of fundamental investors vs price' , '7. display the number of momentum investors vs price','8.display the number of fundamental
investors vs fundamentals','9. display the number of momentuum investors vs fundamentals ','10. momentuum investors who invest at each time step vs time', '11. display the number of fundamental investors VS number of momentum investors who have invested in the stock until now' , '12. display fundamentals vs time' , '13. time VS total number of fundamental investors investing after each time step', '14. display the total number of momentum investors vs time', '15. display the total number of fundamental investors vs price' , '16. display the total number of fundamental investors who invest vs time ' ' '17. display the number of momentuum investors who invest at each step vs fundamentals' , '18. exit')
\% if the user closes the menu box rather than pushing
\% one of the buttons, choice will be 0
while choice == 0
disp(' select an option ')

```
    choice = menu(' Select an options' , '1. readme' , '2. simulate' ,
'3. display the price vs time graph' , '4. display the price vs
fundamentals graph' , '5. display the number of fundamental investors
VS number of momentum investors at this time step', '6. display the
number of fundamental investors vs price', '7. display the number of
momentum investors vs price','8.display the number of fundamental
investors vs fundamentals','9. display the number of momentuum
investors vs fundamentals ','10. momentuum investors who invest at each
time step vs time' , '11. display the number of fundamental investors
VS number of momentum investors who have invested in the stock until
now' , '12. display fundamentals vs time' , '13. time VS total number
of fundamental investors investing after each time step', '14. display
the total number of momentuum investors vs time' , '15. display the
total number of fundamental investors vs price ' ,'16. display the
total number of fundamental investors who invest vs time', '17. display
the number of momentuum investors who invest at each step vs
fundamentals ' , '18. exit' )
end
```


## 12. The analyzes of the output

I will analyze several test cases . I have made available more test cases in the appendix, at the end of the paper.There are even more in the folder. Here I will start with simple cases with small entry data and then proceed to more complex cases and their interpretation .
I developed the application in a test driven way testing the functionality step by step. These test proceed from simple to complex according to the gradual increase in complexity of the application while I was developing it .
The first case I will analyze is a simple case with a financial market populated only by fundamental traders and where the decisions would be taken without regard to probability. The input data is the following with the significance explained above:

5002011111111111020
the bar char describing the values and variation of fundamentals in this simple case:


Next I will present the variation of the price versus time . It may be seen that in this utopic market where there are only fundamental traders the price will follow close the fundamental of companies. There are theories that sustain this opinion like the Efficient market hypothesis. However many consider this theory as not really describing the market. Even in this simple simulation at the next cases more complex it will be seen that price follows the fundamentals but not immediately but on the longer run. It is significant that this result can be seen on a not very complex simulation like this. So let's see now the variation of price in this simple case of a market with only rational agents :


It can be seen from the graph above how as expected the price will increase with the fundamentals because the fundamental traders basing their decision of fundamentals will buy. The input fundamentals increase .

The next graph presents the number of investors investing at each time step and also the price increase. As expected at each time step, because in this simple case there is no probability, all the fundamental investors decide to buy at each decision step. The line with the fundamental investors is at the value of 50 ( a color version can be seen in the archive with more test case - all successful in explaining certain features of the market that I attach!)


In the next graph can be seen how the price follows the fundamentals :


The next graph shows the number of fundamental investors who invest at each time . Here can be seen clearly the confirmation that in this model the number of investors buying at each step is constant for the reason I gave above : all invest at each decision step because the fundamentals rise :


We go now to the next case which is also in the utopic market only with rational , fundamental investors. The input data can be described by the following charts showing the number of steps in time the number and values of the fundamentals and the starting values of the prices and fundamentals :



As you can see also in the graph above in this utopic market the price follows the fundamentals but because this case is a more probabilistic one compared to the previous one it does not follow it in the same exact way as in the deterministic way as in the previous case. At this step I added probabilities in the application and used this case to test .

I show also in the next graphs the correlations between the system parameters :


The previous chart shows the fundamental investors who invested until now. The chart shows how close this number follows the fundamentals. At small level there are probabilities at macro level not, in this small case. Stronger fluctuations will appear in the next cases. Let's see now other correlations in the same case :


As it can be seen from the graph above as it may be expected when the number of investors (fundamentals only) decreases the price also decreases, when the number of investors increases the price also increases.


It can be seen in the graph above the correlation between price and fundamentals . Because this model is already probabilistic the correlation is not perfect but the price follows the trend of the fundamentals .
In the next bar chart may be seen the number of fundamental investors at each decision step :


The next graphs are from a simulation with more steps, 100 decision steps instead of 20 of the previous cases.At this point I still used only fundamental investors . I added gradually also momentum investors and also other features. The input data may be seen from the next charts :



It may be seen in the next graph the correlation of the price and fundamentals. The number of decision steps in greater but the correlation is clear :


From the graph above it can be seen the correlation of the price and fundamentals. It can be seen how the price follows the fundamentals but it actually reflects with a time difference the change in fundamentals. It is very important that on this small scale simulation I obtained this result because this has been observed also in practice and it is considered and important exception to the Efficient Market Hypothesis . The prices trend to react latter to the announcements of increase in the fundamental performance of companies .

The next graph shows that I obtained one more correlation that was to be expected : the investments of the fundamental investors increase when the price of the stock is low and decrease when the price is high. Because the model is probabilistic at this point it can be seen cvasi-random oscillations but the trend is clear :


Let's see a case where the starting price is above the fundamental value : The input data are described by the following charts :


The fundamentals increase, corresponding to a continuous increase in performance of the company but the price will initially decrease because the stock was overpriced initially :


It can be seen above in the 100 step simulation the initial decrease of the price up to the point where the stock price and the fundamental price are equal followed by an increase in price resulted from the interest of the fundamental investors caused by an increased in the performance of the company .
In the next chart can be seen how the price trends to follow the fundamentals but it never really follows perfectly. As I mentioned above this simple model presents an experimental proof of an argument against The efficient market hypothesis resulted from observations in real stock on how the prices follow the announcements of the fundamental results of companies :


Now we will analyze a more complex case , that one of a market with fundamental investors and momentum investors and also probabilistic decisions. The number of fundamental investors I considered here is 150 , the number of momentum investors is 150 and the decision steps are 400 .
The input data is : $1001004001231414-20-12-11551020$
And means that input the starting price of the simulations is the same as the fundamental value.
The graph of the fundamentals vs time is the following :


And contains 10 intervals of variations of the fundamentals corresponding let's say to 10 quarters when the companies publish their reports .
The following is the variation of the price over time :


It can be seen from the graph above that price still follows the fundamentals but because the momentum traders the deviations are greater than in the previous cases. The correlation on the long run of price with the fundamentals can be observed also in the real stock and it is one of the few unquestionable facts among so many different opinions on the stock market so the conclusion are realistic .
The following graphs shows how the price follows the fundamentals in this simulation and this is a realistic feature too .


It may be seen from the next graph the number of fundamental investors after each step and how their decisions determine the price to the greatest extent. However the difference from the price graph shows the significant influence of the 'noise traders', of the momentum traders which cause the noise around the fundamental price :


In this case the momentum investors graph and the fundamental investor graph resembles but it is a difference in time, difference in phase the fundamental investors put the 'smart money' early then the price increases and then after the price increase the momentum investors put the money because the increase in price. This proves experimentally on this system a fact well known in the investment industry. The next is the graph of the fundamental investors . It can be observed that when the price went down towards the end of the simulation the interest of the fundamental investors greatly increased.


The next graph captures in the same image the total number of fundamental and momentum investor after each time step .


The next graph shows that the system captures the correlation between the fundamental and the momentum investors. After the increase in investments from the fundamental investors follows an increase in investment from the momentum investors. This is explained by the investment styles of the two categories of investors. First when the price is low compared to fundamentals the momentum investors buy then in consequence the price has an increase and the momentum investors start to buy too . This is a reason why a difference in time appears between the increase in the intensity of trading activity between the two groups of investors .It can be observed in the next graph how the maximums in blue (the maximum of fundamental investors increase) are followed by high level of activity from the 'noise traders' - the momentum investors ( the maximums in red )


The correlation between the maximum of investment activity of fundamental investors and the maximum of activity of momentum investors is made not directly because the momentum investors do not know exactly when the fundamental investors will invest but is made through the price because the initial rise in price is made by the fundamental investors and that attracts subsequently the momentum investors verifying one more of the hypothesis I presented in this paper. It can also be seen that after the decrease in investment from the fundamental investors will follow a decrease also from the momentum investors. This is made also through the price, not directly, the price decreases after the fundamental investors sell and in consequence also the momentum investors will sell.
From the next graph can be seen that the maxima of the level activity of the momentum investors is after the steepest increase in price. This is of course
explained by the way this category of investors take the investment decision. Also it can be seen that after the time steps when price decreases the interest and the level of activity of the fundamental investors is the smallest. This happens in a probabilistic way in the system I modeled but the trend is clear from the next graph .


Also in the next graph can be seen how the increase in activity from the fundamental investors probabilistically will determine an increase in price.


The activity of the fundamental investors correlates in some degree with the fundamentals. As it can be seen from the next graph the maxima of the investment interest of the fundamental investors is correlated to the fundamental best performance of the economic activity of that company. It depends also on the price of the stock, not only on the performance of the company because even if the performance is very good if the price is very high then it could not be worth the price. If the number or volume of the momentum investors happens to be large it can result in this outcome. However in my model there is a influence of the momentum investors on the oscillations of the system but I did not consider a very big influence so as to put the system in a chaotic path. So as can be seen in the next graph the fundamental investors can follow and actually follow the corporate performance well and decide upon it .


In this regard the momentum traders can complicate the game of the fundamental traders producing high fluctuations in price and high volatility and there are certain cases when the system becomes hard to evaluate fundamentally because these fluctuations produced by noise traders ( momentum traders ). Often the result of noise trading is not only noise but very high fluctuations from high to very high and from very high to very low, called bubbles and crashes .

The next graph summarizes in a single picture the variation of fundamentals, the variation of stock price ( if the level of investment from the noise traders is not very high ) and enforces the conclusions stated above and enforces the idea that the system described in my paper functions well .


From the next graph can be seen how small variations in price can be determined only by a great increase in the number of investors or in the size of the investments they put in the stock. This is very important because once there are no investors in the market who are willing to invest and no resources in the market to sustain that investment then the increase in price cannot be sustained any more and the bubble reverses into a crash . In the next picture it can be seen how small variations of the price are generated by big variation in investment activity :


## 13. Interpretation of the experimental results

- The prices of rational traders will be confined in a smaller interval than the prices of "noise traders".
- the behavior of prices will be confined in an interval if there are rational traders only
- the price will have higher oscillations if there are many noise traders
- the more noise traders, the greater the fluctuations and the more likely a bubble .
- a bubble can be generated in the following way:
step1: first the fundamental economical performance of a company improves
step2: as a logical consequence this attracts rational traders
step3: the flux of rational traders that buy the stock will increase the price
step4: the increase in price will attract the noise traders
step5: the increase in 'noise traders' will increase the demand for the stock and will change the offer-demand equilibrium, the demand dominates the offer .
step6: after the price of the stock increases the market value of the company is greater and greater compared to the fundamental value .
step7: after the price of the company is greater than the fundamental value, the fundamental traders sell their shares .
step8: At the beginning, when the fundamental investors start selling their shares the demand is till greater than the offer. However , the greater the price , the selling starts to dominate the buying. Also , the greater the market value compared to the fundamental value the greater the proportion of noise traders.

Step9: The 'noise traders' are traders who's decision is based on the dynamics of the price. The price will be more and more affected by any small change in price, so a negative feedback mechanism will take place and will produce at some point strong oscillations and possibly the collapse of the price .In this conditions when a small decrease occurs the stop loses of the momentum traders, placed near the current price will be executed and drive the price lower.

Step10: The turning point
The demand will be less and less dominant compared to the offer. The demand and the offer however fluctuate in any conditions and even in moments when the price increases sharply, there will be moments when the demand will be less than the offer . But in a system where the prices are held by the momentum traders, the effects and the probability of a sell-off after a decrease in prices is higher then when a significant number of fundamental traders own the stock. This will generate a turning point and, if after a day of decrease it follows one more day, and then one more the process of sell-off will get speed increasingly as the price drops .

Step11: The price collapses
The drop of the price will continue until the price drops below the fundamental value and it is low enough to attract fundamental traders which see the stock as cheep. The cycle of growth in the price of shares may restart if the fundamental conditions will be proper .

It should be observed that non-Gaussian variations appear in two moments :

1. When the offer is strongly dominated by the demand, so when the stock rises fast
2. When the offer is greater than the demand and the price decreases fast

## 14. Future work in development of this model of artificial financial market

I see the future development of this project as oriented on testing some of the important principles and of some of the hypothesis of finance and mathematical finance .

Market phenomenon to be studied in the future :

- the efficient market hypothesis
- the behavioral finance arguments against the efficient market hypothesis
- the quantitative arguments against the efficient market hypothesis
- the Gaussian and non-gaussian distribution of prices of stocks
- possible development and testing of trading strategies, testing of trading strategies
- testing of methods used and described in books written by famous traders

It would increase the accuracy of the simulation and the range of market phenomenon studied the following possible additions :

- adding simulation of interaction and diffusion of information in networks of investors
- testing the impact of news on trading
- testing the impact of system structural crises
- adding more types of stocks
- adding more types of economic branches
- adding a more complex description of fundamental performance of companies
- using real stock data to decide on the values of parameters
- using machine learning to optimize the systems and decide on parameters
- using system theory to model a stochastic dynamic system with control mechanism , feedback, estimation an so on
- calculating more advanced statistics on the stock processes
- adding a mechanism for price formation closer to the industrial system and closer to the way clients and brokerage operation works .
- adding real time stock data
- adding information from the databases with the fundamentals of companies who's shares are traded on the stock exchange .


## 15.Bibliography

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## appendix :

## the testing and developing process and a more complete list of test cases I used

## general system development and testing <br> strategy :

- building the system and testing after each function ( called test-driven development )
- implementing first the functionality for fundamental investors
- implementing first decisions using deterministic rules and adding probabilistic decisions after that
- changing parameters to test the sensitivity of the system to changes in parameters
- starting with smaller cases, determined here by the number of fundamental investors, the number of momentum investors and the number of stages in simulation. I tested here from 10 stages to 1000 and many of the complex cases are about 400 stages .
- selection of test cases to reflect different decision rules and different combinations of input data and decision rules
the first step in testing : the case of a market with only fundamental investors :
case 1 : fundamental investors; the fundamentals have a decreasing trend ; the starting price is above the fundamental price
case 1.1 use 20 steps in simulation
case 1.1.a under probability :
the input data: 5002012311 -13-1121020
the significance of he input is presented above
next are the graphs that show the correlations and prove the function of the system in more and more complex cases. The significance of the graph appears on display .




fundamentals vs time - blue / momentuum investors who invest at each time step vs time - red

price vs time - blue | fundamental investors who invest at each time step vs time - red




## case 1.2 used 100 steps in simulation




time vs number of fundamental investors investing at each step vs momentuum investors


fundamentals vs time - blue | fundamental investors who invest at each time step vs time - red



fundamentals vs time - blue $/$ momentuum investors who invest at each time step vs time - red


case 2 : fundamental investors ; the fundamentals have a decreasing trend ; the starting price is above the fundamental price
the input data : $500100-1-1-0.5-0.5-1-1.2-1.1-1-0.5-13020$
( the significance of the input data is described in above )


fundamentals vs time - blue / fundamental investors who invest at each time step vs time - red

fundamentals vs time - blue / momentuum investors who invest at each time step vs time - red



fundamentals vs time - blue I momentuum investors who invest at each time step vs time - red







Case 3 : fundamental and momentum investors, 150 fundamental investors and 150 momentuum investors, 400 time steps in simulation
In the system there are fundamental investors and momentum investors. The fundamentals are :



time vs number of fundamental investors investing at each step (b) ws momentuum investors (r)











The graph above contains also the change in fundamentals and the change in price.




