On the Controversy around Daganzo's Requiem for and Aw-Rascle's Resurrection of Second-Order Traffic Flow Models

Definitions

- Second Order Traffic Flow Model
  - Continuity equation :

$$\frac{\partial \rho(x,t)}{\partial t} + V(x,t)\frac{\partial \rho(x,t)}{\partial x} = -\rho(x,t)\frac{\partial V(x,t)}{\partial x}$$

  - Dynamic velocity equation :

$$\frac{\partial V(x,t)}{\partial t} + V(x,t)\frac{\partial V(x,t)}{\partial x} = -\frac{1}{\rho}\frac{\partial P_1(\rho,V)}{\partial \rho}\frac{\partial \rho(x,t)}{\partial x} - \frac{1}{\rho}\frac{\partial P_2(\rho,V)}{\partial V}\frac{\partial V(x,t)}{\partial x} + \frac{V_o(\rho,V) - V(x,t)}{\tau}$$

- Group velocity

- Principle of causality

What is the problem ?

- Daganzo's Requiem :
  - Second-order models suffer from several inconsistencies
  - They always exhibit one group velocity greater than vehicles velocities :

    the traffic is determined by what is happening behind it!!!

- How can we address this issue ? Do group velocities greater than vehicle speeds really imply violation of causality ?

Derivation of characteristic speed

- Linearized equations
  - Evolution of small perturbation of equilibrium traffic

$$\delta\rho(x,t) = \rho(x,t) - \rho_{\mathrm{e}}$$

$$\delta V(x,t) = V(x,t) - V_{\mathrm{e}}$$

- Analogy with sound waves
  - Different eigenmodes, characterized by

wave number κ frequencies $\omega_l(\kappa)$ growth rates $\lambda_l(\kappa)$

  - Associated characteristic speed, group velocity and phase velocity

$$V_{\mathrm{e}} + \frac{1}{2\rho_{\mathrm{e}}}\frac{\partial P_2}{\partial V} \pm \sqrt{\frac{1}{4{\rho_{\mathrm{e}}}^2}\left(\frac{\partial P_2}{\partial V}\right)^2 + \frac{\partial P_1}{\partial \rho}} \qquad = \frac{\partial \omega_l(\rho_{\mathrm{e}},\kappa)}{\partial \kappa} \qquad \frac{\tilde{\omega}_{\pm}(\rho_{\mathrm{e}},\kappa)}{\kappa}$$

Two examples

- Tunable parameters : τ, $P_1(\rho, V)$, $P_2(\rho, V)$

- Aw's and Rascle's model
  - $\tau \to \infty$ $\quad \dfrac{\partial P_1(\rho,V)}{\partial \rho} = 0 \quad \dfrac{\partial P_2(\rho,V)}{\partial V} = -\gamma\rho(x,t)^{\gamma+1}$
  - Characteristic speeds : $\boxed{V - \gamma\rho(x,t)^{\gamma} \qquad V}$

- Payne's model
  - $V_o(\rho,V) = V_e(\rho)$ $\quad \dfrac{\partial P_1(\rho,V)}{\partial \rho} = \dfrac{1}{2\tau}\left|\dfrac{dV_e(\rho)}{d\rho}\right| \quad \dfrac{\partial P_2(\rho,V)}{\partial V} = 0$
  - Characteristic speeds : $\boxed{V_e \mp \rho_e \left|\dfrac{dV_e(\rho_e)}{d\rho}\right|}$

Discussion (I)

- Variance of speed

  - V_e is a mean value of vehicle speed. In practice : each vehicle has a different velocity

    fluctuations ΔV around the mean value V_e

  - With certain constraints, we find that $V_g < V_e + \Delta V$

    some vehicle go faster than V_g

ex: Payne's model, choose :

$$V_e(\rho) = V^0 \left(1 - \frac{\rho}{\rho_{\mathrm{jam}}}\right)$$

Discussion (II)

- Stability analysis
  - The greater characteristic speed is associated with negative eigenvalue of linear equations
  - Such a mode decays quickly

difficult to observe this mode in reality, does not emerge by itself

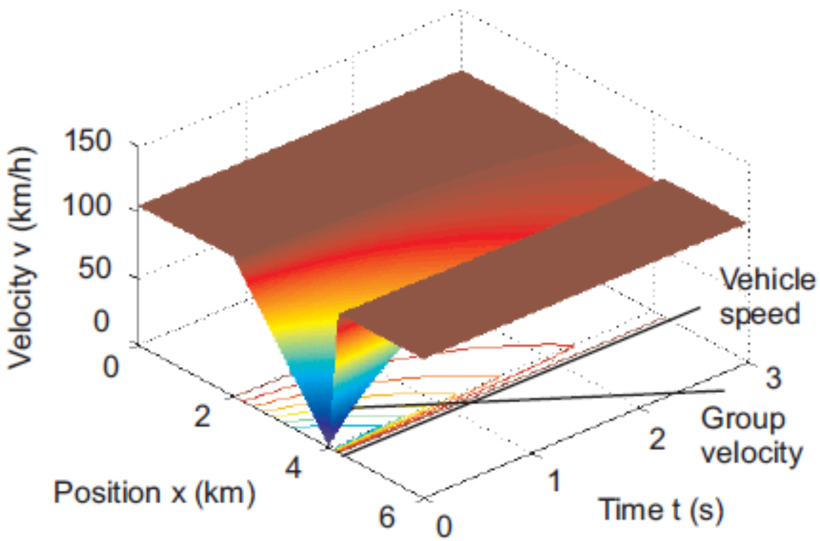
Microscopic model

- Consider each vehicle speed and position, interaction only with the leader

$$\frac{dv_i}{dt} = \frac{v_o(d_i(t)) - v_i(t)}{\tau}$$

$$\frac{dd_i}{dt} = v_{i-1}(t) - v_i(t)$$

- Under specific choice of intial conditions, same phenomenon



not a specific feature of second-orders models

Conclusion

- Most macroscopic traffic models have a characteristic speed faster than average velocity
- However, they do not constitute a theoretical inconsistency
- Interpretation of group velocity as speed of information transmission can be misleading