MERITOCRATIC MATCHING & PUBLIC GOODS GAMES

HEINRICH H. NAX (HNAX@ETHZ.CH)

SEPTEMBER 10, 2014

HOMO SOCI VS ECON WORKSHOP

SOURCES

The material presented in this talk is based on joint work with

- *Stefano Balietti*
- *Dirk Helbing*
- *Ryan Murphy*

Results can be divided into two parts

- *theory*
- *experiments*

Please see
http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2404280

WHAT IS "MERITOCRATIC MATCHING"?

WHAT DOES IT DO/ MEAN IN "PUBLIC GOODS GAMES"?

"MERITOCRACY": DEFINITIONS

Def.: "rule by those with merit/ rule rewarding merit"

- old concept with a surprisingly new name (Young 1958)
- present in early modern societies including China, Greece, Rome
- examples include selection of officials/ councilmen, military reward/ promotion schemes and access to education
- proposed by thinkers such as Confucius, Aristotle and Plato

Criticism: as identified, for example, in the book by Arrow, Bowles and Durlauf (2000) is the inherent

- **inequality-efficiency trade-off** (e.g. education)

“PUBLIC GOODS GAMES”: VOLUNTARY CONTRIBUTIONS

- **every player i chooses whether to contribute ($c_i = 1$) or not ($c_i = 0$)**
- **given contributions, players are matched (how remains to be specified) into groups of fixed size *s***
- **given contributions in each group G, for a *marginal per-capita rate of return* ($mpcr$) $r/s \in (1/s, 1)$, a public good is provided and its return split equally so that i's payoff is**

$$u_i(c) = (1 - c_i) + \sum_{\substack{j \in G_i: \\ i \in G_i}} mpcr * c_j$$

MERITOCRATIC MATCHING

1. **actual contributions c_i are chosen by players**
2. **Gaussian *noise* with mean 0 and variance $1/\beta$ added to actual contributions**
3. **β is the index of meritocracy in the system**
4. **players are ranked by *noised contributions***
5. **groups form according to the ranking (with random tie-breaking)**
6. **payoffs materialize based on actual contributions**

$\boldsymbol{\beta}$-MERITOCRATIC MATCHING

$\boldsymbol{\beta} \to \mathbf{0}$ ⟶ $\boldsymbol{\beta} \to \infty$

No meritocracy

Intermediate level of meritocracy

Perfect meritocracy

RELATED WORK

"STANDARD" VOLUNTARY CONTRIBUTIONS MECHANISMS

(here: $\beta \to 0$)

- **basis: standard voluntary contributions game (Marwell and Ames 1979, Isaac et al. 1985)**
- **group matching: random group (re-)matching (Andreoni 1988)**

Outcome**: the only equilibrium is all free-ride.**

GROUP-BASED MECHANISM

(here: $\boldsymbol{\beta} \rightarrow \infty$)

- **basis: standard voluntary contributions game (Marwell and Ames 1979, Isaac et al. 1985)**
- **group matching: groups form according to rank (Gunnthorsdottir et al. 2010)**

Outcome: **if the $mpcr$ is high enough, a new equilibrium in pure strategies emerges where the majority contributes and a small minority free-rides (Gunnthorsdottir et al. 2010, Theorem 1).**

FURTHER

(comparable to general β)

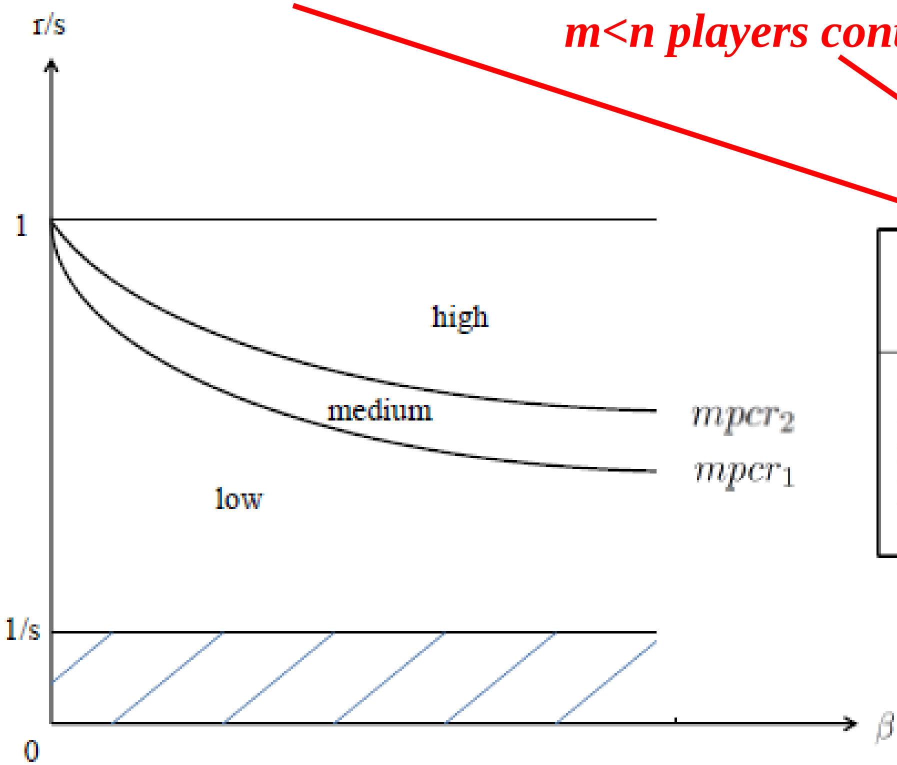
- **preference-assortative matching**
  (Alger & Weibull 2013, Jensen & Rigos 2014)
- **local reproduction/ local interaction**
  (Hamilton 1964, Grund et al. 2013)

Outcome: **“if process is sufficiently assortative, outcomes related to our high equilibria can emerge”…**

MERITOCRATIC MATCHING: THEORY

theoretical results i: NASH EQUILIBRIA
(see NAX ET AL. 2014, PROPOSITIONS 6-10)

If the *mpcr* is high enough, there may be new Nash equilibria:



n players free-ride → 1^0

m<n players contribute → 1^m

every player contributes with probability p>0 → 1^p

	1^0	1^m	1^p
low	✔	✗	✗
medium	✔	✔	✗
high	✔	✔	✔

SOME BEST REPLY EXAMPLES

Suppose $\mathbf{n = 8,\ \boldsymbol{s} = 4,\ mpcr = 0.5,\ \boldsymbol{\beta} \to \infty;}$

i.e. two groups are matched under perfect merticracy

EXAMPLE 1:

Others:

all contribute 0

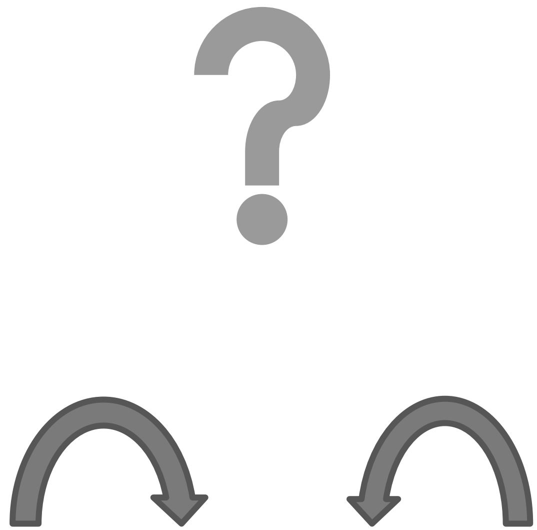
You:

What is the best reply?

EXAMPLE 1: BEST REPLY

CONTRIBUTE (1)	FREE-RIDE (0)
1 > payoff 10	0
0	0
0	0
0	0
0	0
0	0 > payoff 20
0	0
0	0

EXAMPLE 2:



Others:

all contribute 1

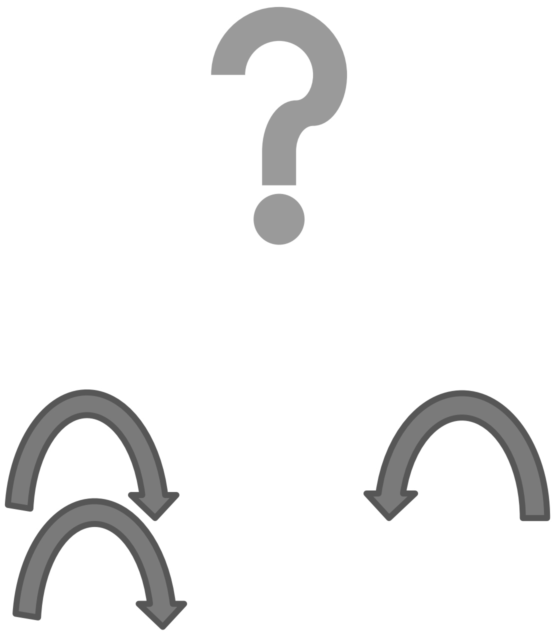
You:

What is the best reply?

EXAMPLE 2: BEST REPLY

CONTRIBUTE (1)	FREE-RIDE (0)
1	**1**
1	**1**
1 > payoff 40	**1**
1	**1**
___	___
1	**1**
1	**1**
1	**1**
1	**0 > payoff 50**

EXAMPLE 3:



Others:

4 contribute 0

3 contribute 1

You:

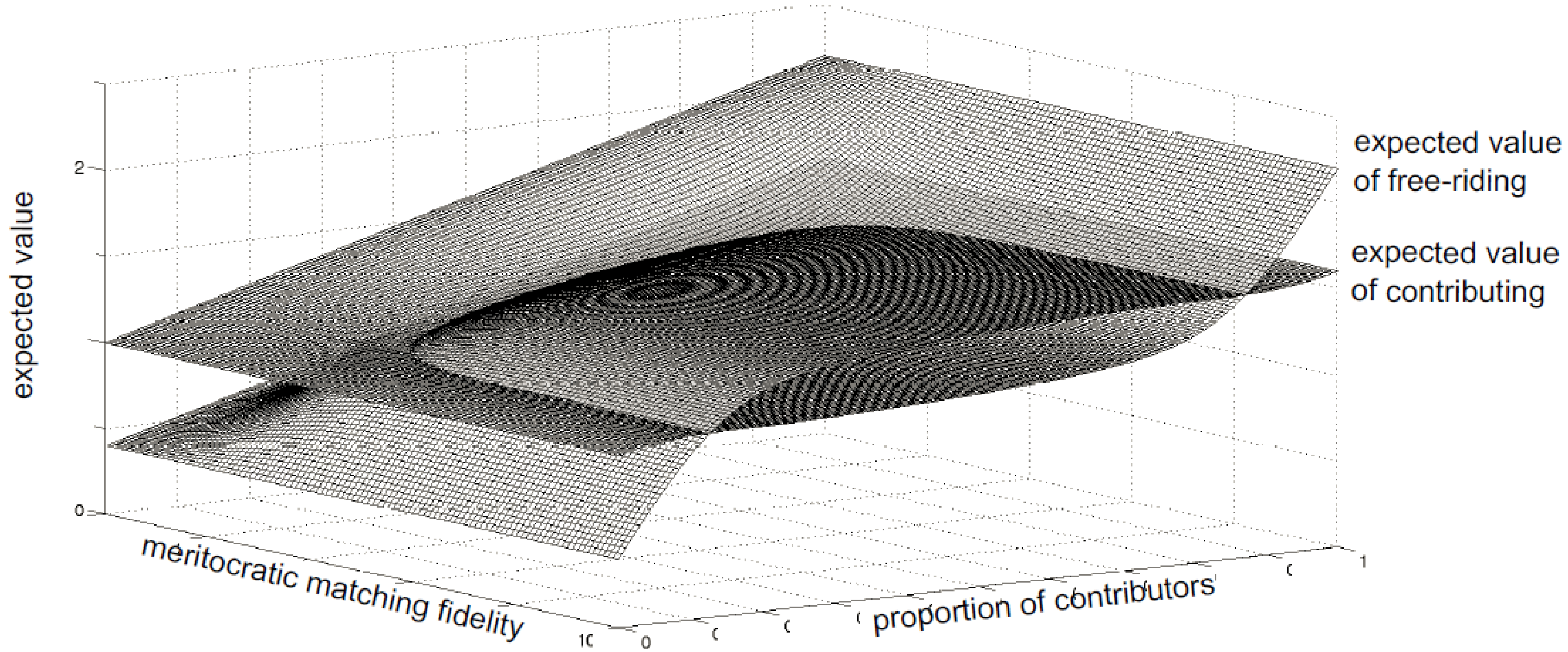
What is the best reply

EXAMPLE 3: BEST REPLY

CONTRIBUTE (1)	FREE-RIDE (0)	
1	**1**	**1**
1	**1**	**1**
1 > payoff 40	**1**	**1**
1	**0 > payoff 50**	**0**
0	**0**	**0**
0	**0**	**0 > payoff 20**
0	**0**	**0**
0	**0**	**0**
	p=0.2	**p=0.8**

EXPECTATION (0) = 26 < 40

BEST REPLY SPACE: “NEW EQUILIBRIA”



EVOLUTIONARY DYNAMICS

- **suppose the game is repeated at time steps t=1,2,3,…**
- **consider**
  - either **replicator dynamics** (e.g., as in Weibull 1993, Helbing 1996)
  - or **perturbed best reply dynamics** with a fixed population (e.g., as in Young 1993)

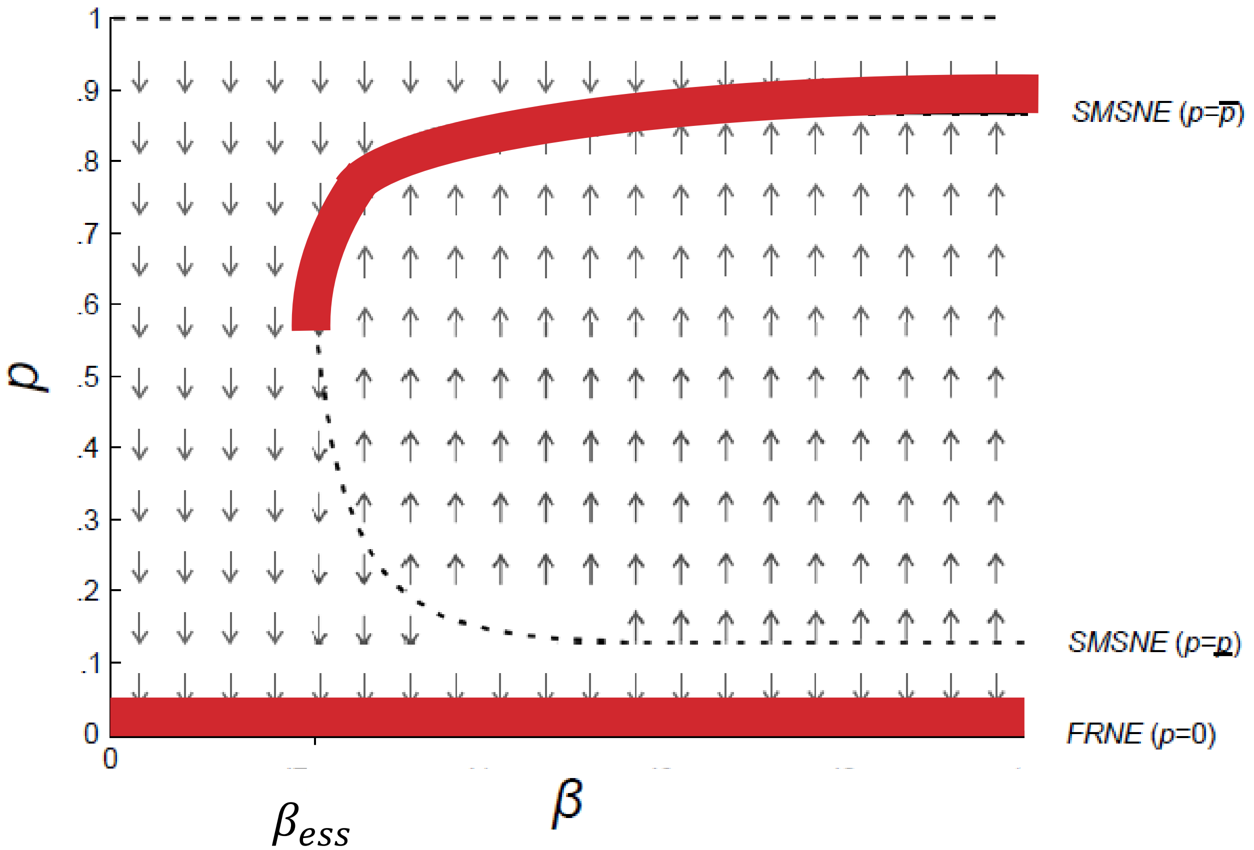
REPLICATOR EQUATION

Suppose the proportion of contributors evolves according to the following equation:

$$\partial p / \partial t = (1-p)p\left(\mathbf{E}\left[\phi_i(1|1_p)\right] - \mathbf{E}\left[\phi_i(0|1_p)\right]\right)$$

theoretical results ii.a: STABILITY -ESS-

(see NAX ET AL. 2014, LEMMA 1)



PERTURBED BEST REPLY

Suppose now the population does not grow. Instead the same *n* agents continue playing the game ad infinitum.

Suppose that, each period,

- **each agent plays a *myopic best reply* to the previous-period actions of the n-1 other players**

with probability 1-ε

- **and the other action**

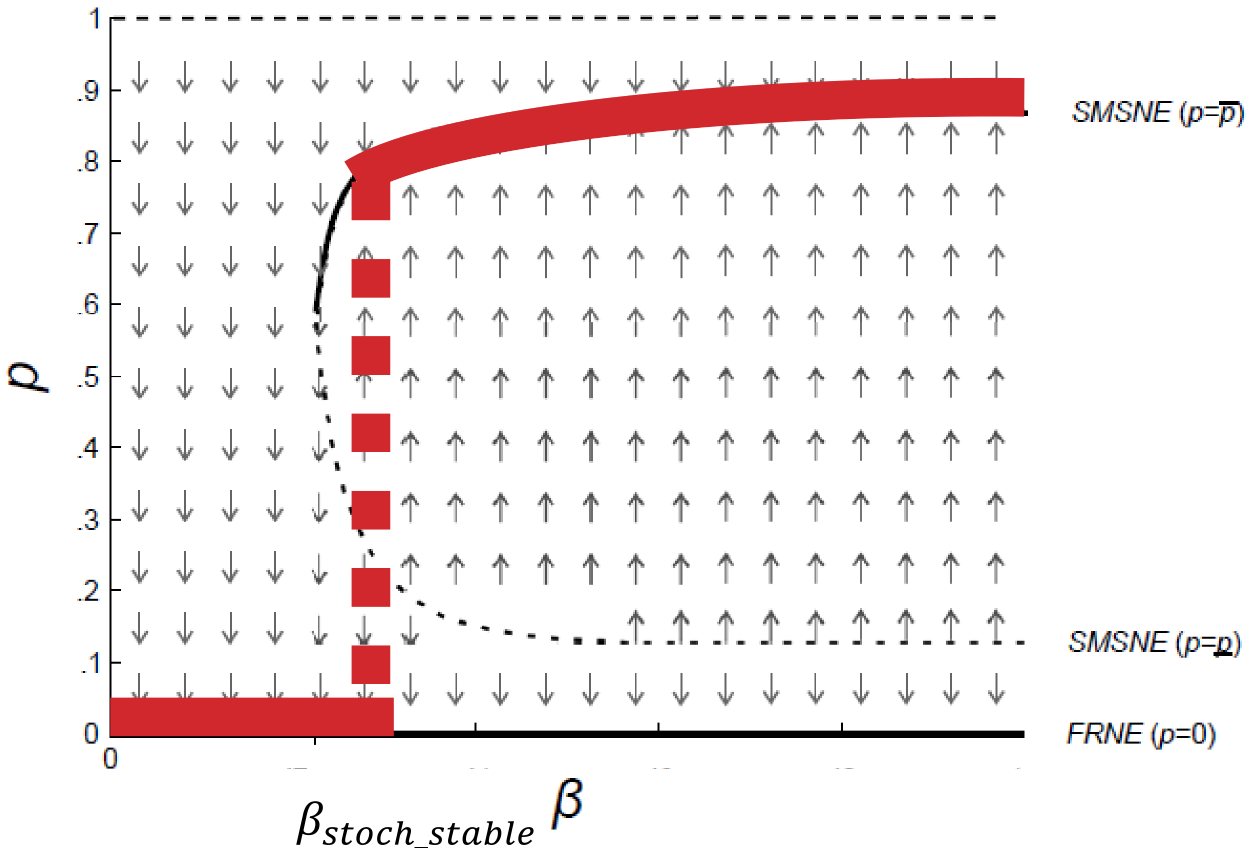
with probability ε

ε being something like the "error rate"

STOCHASTIC STABILITY

Definition: **A state is *stochastically stable* (Foster and Young 1990) if the stationary distribution as ε goes toward 0 places positive weight on that state.**

theoretical results ii.b:
STABILITY -STOCHASTIC STABILITY-
(see NAX ET AL. 2014, LEMMA 3)



theoretical results iii: WELFARE
(see NAX ET AL. 2014, PROPOSITION 3)

Say *social welfare* given *inequality aversion* parameter $e \in [0, \infty)$ is

$$W_e(u) = \frac{1}{n(1-e)} * \sum_{i \in N} u_i^{1-e}$$

(When $e = 1$, assume $W_e(u) = \frac{1}{n} \prod_{i \in N} u_i$, i.e. the Nash product.)

$W_e(u)$ is a variant of the function by Atkinson (1970) nesting

- Benthiam welfare   if $e = 0$
- Rawlsian welfare   if $e \to \infty$

***Result:* social planner setting β will set $\beta = 0$ only if very inequality averse, else $\beta = \beta_{stoch\_stable}$.**

***Reason:* depending on e, the near-efficient pure-strategy Nash equilibrium or the free-riding equilibrium is preferred (efficiency-inequality trade-off).**

WELFARE ILLUSTRATIONS

$W_e(u)$ with $e > 10.3$ "HIGH" EQUILIBRIUM		$W_e(u)$ with $e < 10.3$ FREE-RIDING EQUILIBRIUM
0	0.0	0
0	0.2	0
0	0.4	0
0	0.6	0
13 14 $(c_i = 1)$ 2	0.8	0
0	1.0	16 $(c_i = 0)$ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
0	1.2	0
0	1.4	0
1 2 3 4 5 6 7 8 9 10 11 12 $(c_i = 1)$ 12	1.6	0
15 16 $(c_i = 0)$ 2	1.8	0
24.4	efficiency	16

With $e > 10.3$, the social planner requires efficiency gains of *more than twice the amount lost by any player* to compensate for the additional inequality…

MERITOCRATIC MATCHING: EXPERIMENTS

Experiments: SET-UP i/ii

- **Experiments were conducted in May/ June 2014 @ DeSciL (involving 192 subjects in 12 sessions)**

- **In each session, 16 players played two of our games**
- **The $mpcr$ was always 0.5 and the group size always 4**
- **The budget was 20 coins each round**
- **The game was repeated 40 rounds**
- **Players received full instructions and (anonymous) feedback about previous-period play**
- **Play was incentivized with real money (one coin=0.01 CHF)**

- **Games differed w.r.t. variance levels: $0, 3, 20,$ or ∞.**
  (*Note*: when 20 or ∞, the near-efficient equilibrium does not exist.)

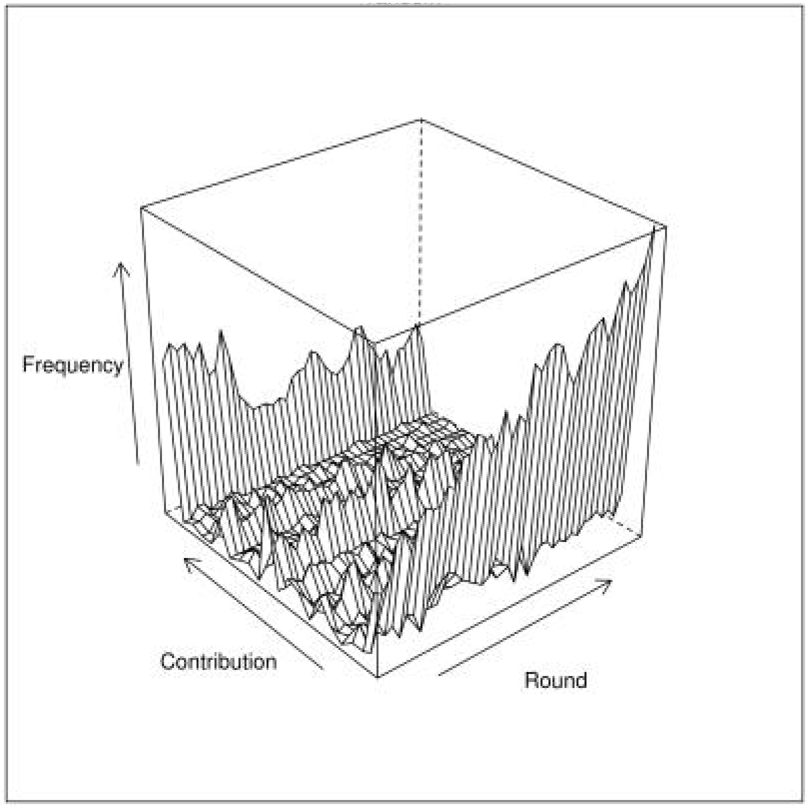
Experiments: SET-UP ii/ii

- **Each player played two games; each one with a different variance level**
- **All possible ordered pairs of variance levels were played and made up a separate session**
- **A somewhat *hybrid* design:**
  between-subject/ within-subject
- **Each player experienced either a meritocracy increase or a meritocracy decrease**
- **12 sessions = 6 possible variance pairs * 2 orders each**

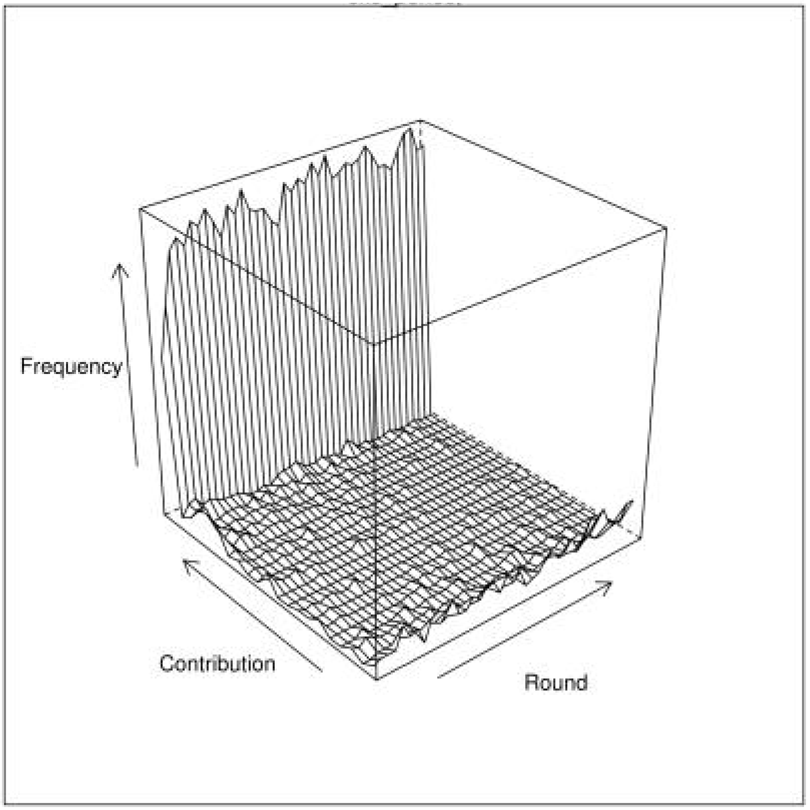
EXPERIMENTAL EVIDENCE

CONTRIBUTIONS 1:

RANDOM

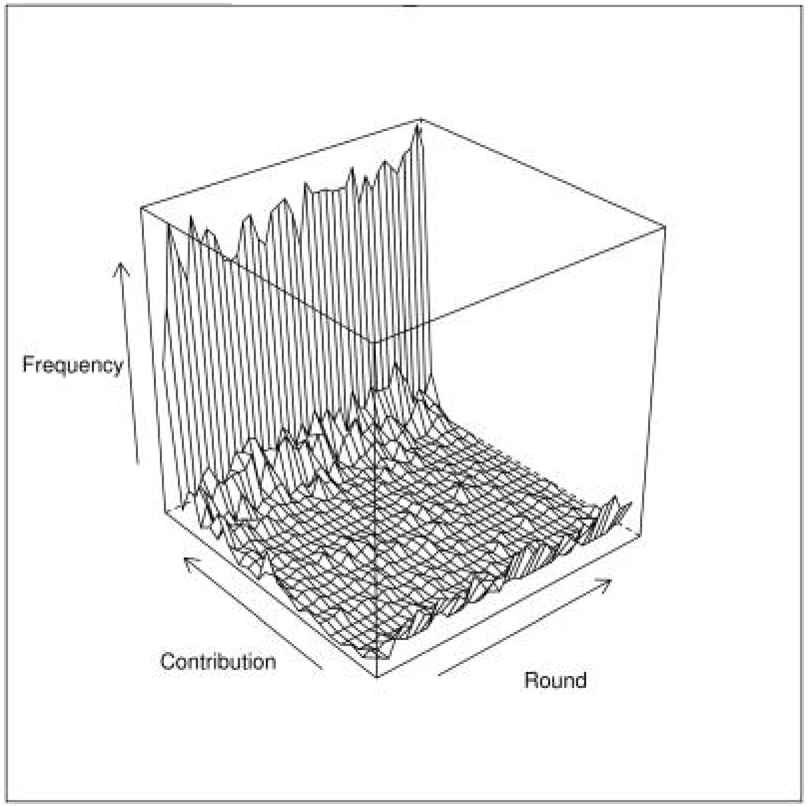


PERFECT

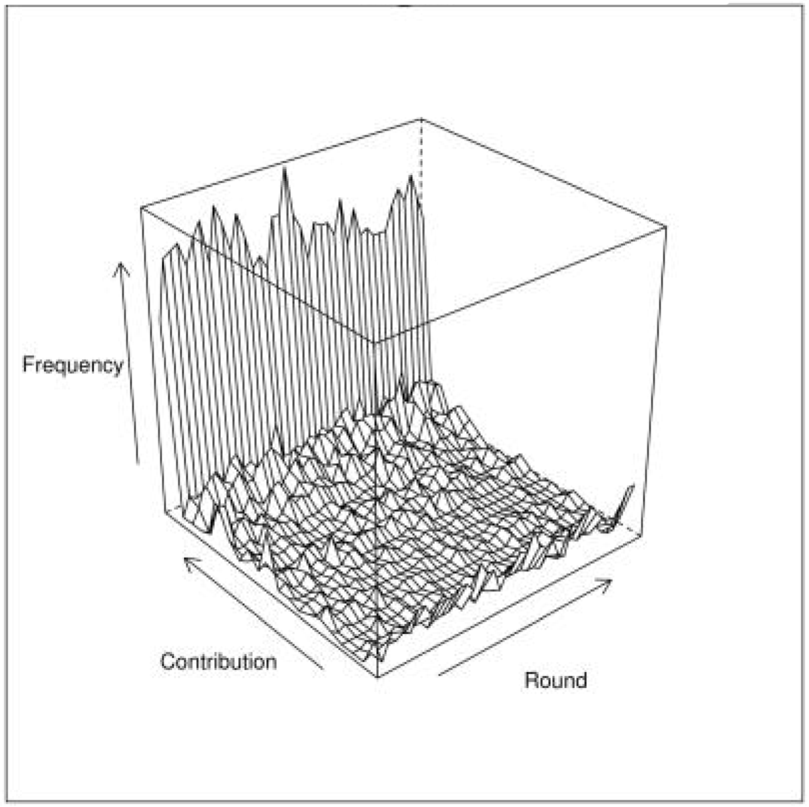


CONTRIBUTIONS 2:

Var3

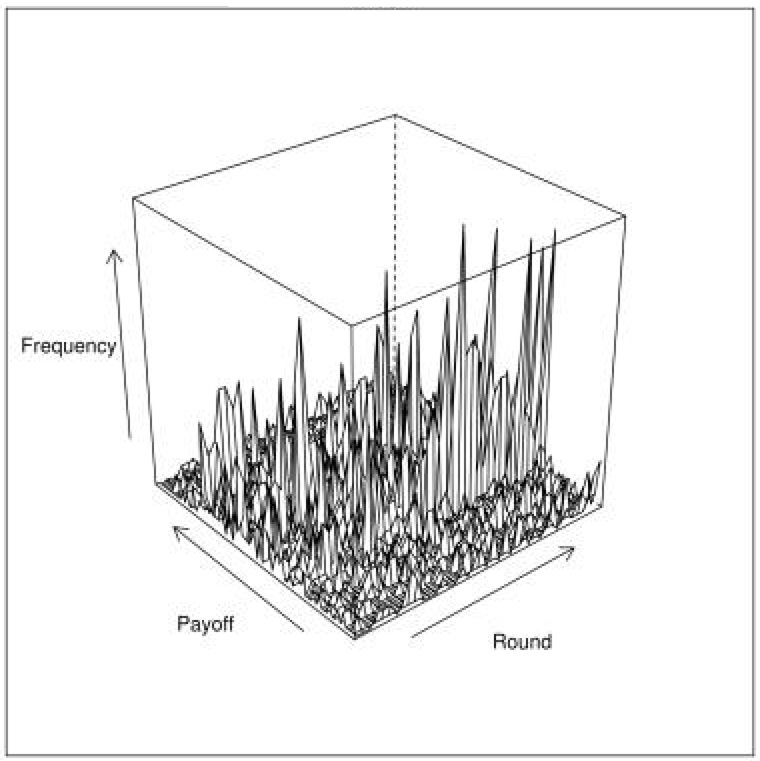


Var20

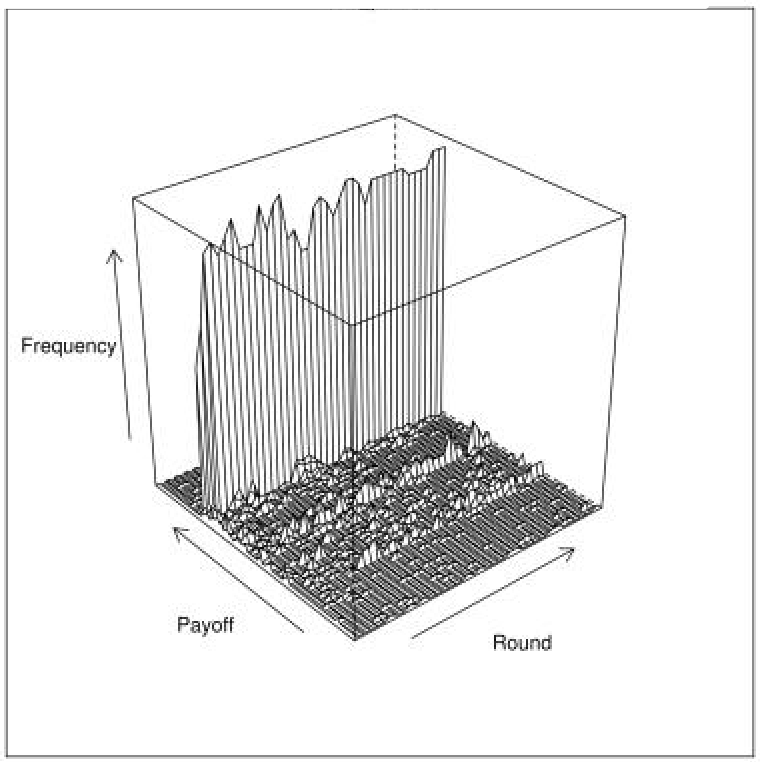


PAYOFFS 1:

RANDOM

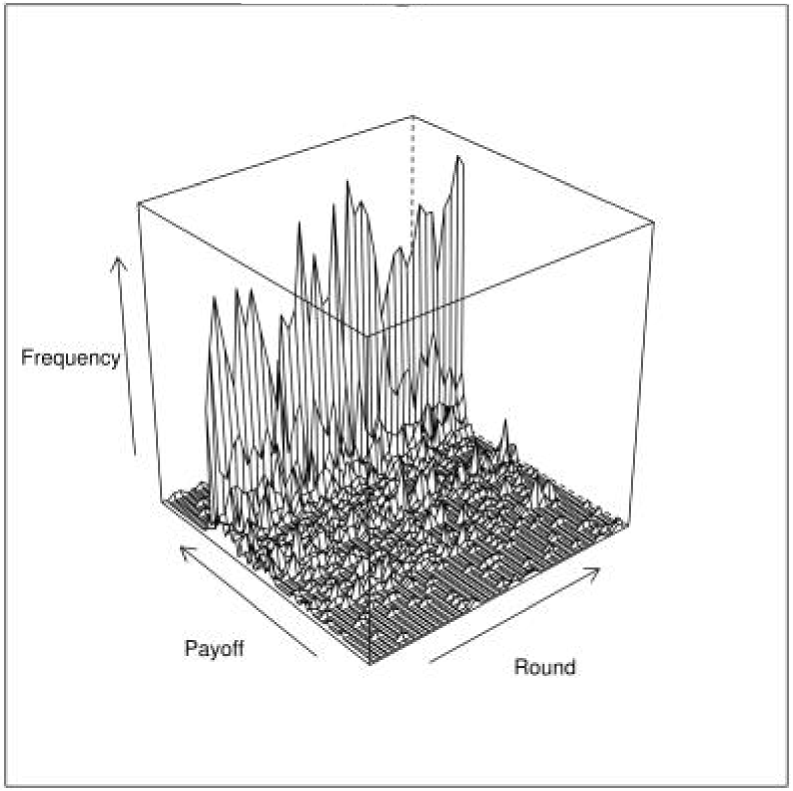


PERFECT

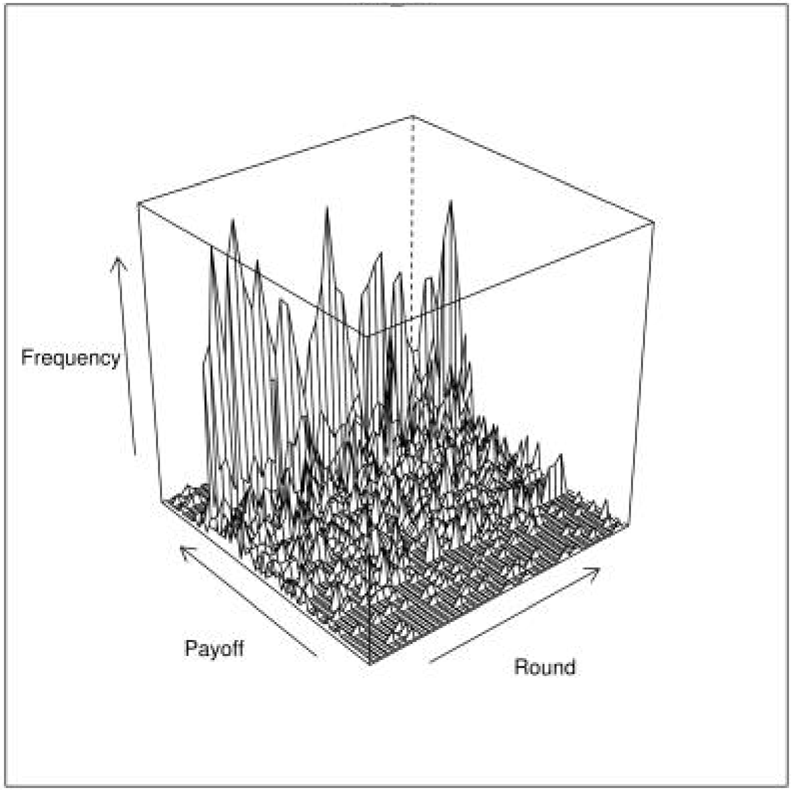


PAYOFFS 2:

Var3



Var20



CONCLUSION

- **Moderate levels of meritocracy in matching help enable new, near-efficient equilibria**
- **New equilibria are indeed also more stable if the mechanism is sufficiently meritocratic**
- **New equilibria are typically preferable w.r.t. social welfare even with substantial inequality aversion**

THEORY LESSONS

- **In practice, a “hint” of meritocracy may prove sufficient to reach more efficient outcomes with high contributions**
- **Realized inequality is substantially lower in higher meritocracy regimes**

EXPERIMENTAL EVIDENCE

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