PEOPLE GET SEPARATED along many lines and in many ways. There is segregation by sex, age, income, language, religion, color, personal taste, and the accidents of historical location. Some segregation results from the practices of organizations. Some is deliberately organized. Some results from the interplay of individual choices that discriminate. Some of it results from specialized communication systems, like languages. And some segregation is a corollary of other modes of segregation: residence is correlated with job location and transport.

If blacks exclude whites from their church, or whites exclude blacks, the segregation is organized; and it may be reciprocal or one-sided. If blacks just happen to be Baptists and whites Methodists, the two colors will be segregated Sunday morning whether they intend to be or not. If blacks join a black church because they are more comfortable among their own color, and whites a white church for the same reason, undirected individual choice can lead to segregation. And if the church bulletin board is where people advertise rooms for rent, blacks will rent rooms from blacks and whites from whites because of a communication system that is connected with churches that are correlated with color.

Some of the same mechanisms segregate college professors. The college may own some housing, from which all but college staff are excluded. Professors choose housing commensurate with their incomes, and houses are clustered by price while professors are clustered by income. Some professors prefer an academic neighborhood; any differential in professorial density will cause them to converge and increase the local density, and attract more professors. And house-hunting professors learn about available housing from colleagues and their spouses, and the houses they learn about are naturally the ones in neighborhood where professors already live.

The similarity ends there, and nobody is about to propose a commission to desegregate academics. Professors are not much missed by those they escape from in their residential choices. They are not much noticed by those they live among, and, though proportionately concentrated, are usually a minority in their neighborhood. While indeed they escape classes of people they would not care to live among, they are more conscious of where they do live than of where they don't, and the active choice is more like congregation than segregation, though the result may not be so different.

This chapter is about the kind of segregation—or separation, or sorting—that can result from discriminatory individual behavior. By "discriminatory" I mean reflecting an awareness, conscious or unconscious, of sex or age or religion or color or whatever the basis of segregation is, an awareness that influences decisions on where to live, whom to sit by, what occupation to join or to avoid, whom to play with, or whom to talk to. It examines some of the *individual* incentives and individual perceptions of difference that can lead *collectively* to segregation. It also examines the extent to which inferences can be drawn from actual collective segregation about the preferences of individuals, the strengths of those preferences, and the facilities for exercising them.

The main concern is segregation by "color" in the United States. The analysis, though, is so abstract that any twofold distinction could constitute an interpretation—whites and blacks, boys and girls, officers and enlisted men, students and faculty. The only requirement of the analysis is that the distinction be twofold, exhaustive, and recognizable. (Skin color, of course, is neither dichotomous nor even unidimensional, but by convention the distinction is nearly twofold, even in the United States census.)

At least two main processes of segregation are outside this analysis. One is organized action—legal or illegal, coercive or merely exclusionary, subtle or flagrant, open or covert, kindly or malicious, moralistic or pragmatic. The other is the process, largely but not entirely economic, by which the poor get separated from the rich, the less educated from the more educated, the unskilled from the skilled, the poorly dressed from the well dressed—in where they work and live and eat and play, in whom they know and whom they date and whom they go to school with. Evidently color is correlated with income, and income with residence; so even if residential choices were color-blind and unconstrained by organized discrimination, whites and blacks would not be randomly distributed among residences.

It is not easy to draw the lines separating "individually motivated" segregation from the more organized kind or from the economically induced kind. Habit and tradition are substitutes for organization. Fear of sanctions can coerce behavior whether or not the fear is justified, and whether the sanctions are consensual, conspiratorial, or dictated. Common expectations can lead to concerted behavior.

The economically induced separation is also intermixed with discrimination. To choose a neighborhood is to choose neighbors. To pick a neighborhood with good schools, for example, is to pick a neighborhood of *people* who want good schools. People may furthermore rely, even in making economic choices, on information that is color-discriminating; believing that darker-skinned people are on the average poorer than lighter-skinned, one may consciously or unconsciously rely on color as an index of poverty or, believing that others rely on color as an index, adopt their signals and indices accordingly.

For all these reasons, the lines dividing the individually motivated, the collectively enforced, and the economically induced segregation are not clear lines at all. They are furthermore not the only mechanisms of segregation. Separate or specialized communication systems—especially distinct languages—can have a strong segregating influence that, though interacting with the three processes mentioned, is nevertheless a different one.

Individual Incentives and Collective Results

Economists are familiar with systems that lead to aggregate results that the individual neither intends nor needs to be aware of, results that sometimes have no recognizable counterpart at the level of the individual. The creation of money by a commercial banking system is one; the way savings decisions cause depressions or inflations is another.

Biological evolution is responsible for a lot of sorting and separating, but the little creatures that mate and reproduce and forage for food would be amazed to know that they were bringing about separation of species, territorial sorting, or the extinction of species. Among social examples, the coexistence or extinction of second languages is a phenomenon that, though affected by decrees and school curricula, corresponds to no conscious collective choice.

Romance and marriage, as emphasized in Chapter 1, are exceedingly individual and private activities, at least in this country, but their genetic consequences are altogether aggregate. The law and the church may constrain us in our choices, and some traditions of segregation are enormously coercive; but, outside of royal families, there are few marriages that are part of a genetic plan. When a short boy marries a tall girl, or a blonde a brunette, it is no part of the individual's purpose to increase genetic randomness or to change some frequency distribution within the population.

Some of the phenomena of segregation may be similarly complex in relation to the dynamics of individual choice. One might even be tempted to suppose that some "unseen hand" separates people in a manner that, though foreseen and intended by no one, corresponds to some consensus or collective preference or popular will. But in economics we know a great many macro-phenomena, like depression and inflation, that do not reflect any universal desire for lower incomes or higher prices. The same applies to bank failures and market

crashes. What goes on in the "hearts and minds" of small savers has little to do with whether or not they cause a depression. The hearts and minds and motives and habits of millions of people who participate in a segregated society may or may not bear close correspondence with the massive results that collectively they can generate.

A special reason for doubting any social efficiency in aggregate segregation is that the range of choice is often so meager. The demographic map of almost any American metropolitan area suggests that it is easy to find residential areas that are all white or nearly so and areas that are all black or nearly so but hard to find localities in which neither whites nor nonwhites are more than, say, three-quarters of the total. And, comparing decennial maps, it is nearly impossible to find an area that, if integrated within that range, will remain integrated long enough for a couple to get their house paid for or their children through school.

Some Quantitative Constraints

Counting blacks and whites in a residential block or on a baseball team will not tell how they get along. But it tells something, especially if numbers and ratios matter to the people who are moving in or out of the block or being recruited for the team. With quantitative analysis there are a few logical constraints, analogous to the balance-sheet identities in economics. (Being logical constraints, they contain no news unless one just never thought of them before.)

The simplest constraint on dichotomous mixing is that, within a given set of boundaries, not both groups can enjoy numerical superiority. For the whole population the numerical ratio is determined at any given time; but locally, in a city or a neighborhood, a church or a school or a restaurant, either blacks or whites can be a majority. But if each insists on being a local majority, there is only one mixture that will satisfy them—complete segregation.

Relaxing the condition, if whites want to be at least three-fourths and blacks at least one-third, it won't work. If whites want to be at least two-thirds and blacks no fewer than one-fifth, there is a small range of mixtures that meet the conditions. And not everybody can be in the mixtures if the overall ratio is outside the range.

In spatial arrangements, like a neighborhood or a hospital ward, everybody is next to somebody. A neighborhood may be 10 percent black or white; but if you have a neighbor on either side, the minimum nonzero percentage of opposite color is 50. If people draw their boundaries differently we can have everybody in a minority: at dinner, with men and women seated alternately, everyone is outnumbered two to one locally by the opposite sex but can join a three-fifths majority if he extends his horizon to the next person on either side.

Separating Mechanisms

The simple mathematics of ratios and mixtures tells us something about what outcomes are logically possible, but tells us little about the behavior that leads to, or that leads away from, particular outcomes. To understand what kinds of segregation or integration may result from individual choice, we have to look at the processes by which various mixtures and separations are brought about. We have to look at the incentives and the behavior that the incentives motivate, and particularly the way that different individuals comprising the society impinge on each other's choices and react to each other's presence.

There are many different incentives or criteria by which blacks and whites, or boys and girls, become separated. Whites may simply prefer to be among whites and blacks among blacks. Alternatively, whites may merely avoid or escape blacks and blacks avoid or escape whites. Whites may prefer the company of whites, while the blacks don't care. Whites may prefer to be among whites and blacks also prefer to be among whites, but if the whites can afford to live or to eat or to belong where the blacks cannot afford to follow, separation can occur.

Whites and blacks may not mind each other's presence, may even prefer integration, but may nevertheless wish to avoid minority status. Except for a mixture at exactly 50:50, no mixture will then be self-sustaining because there is none without a minority, and if the minority evacuates, complete segregation occurs. If both blacks and whites can tolerate minority status but place a limit on how small the minority is—for example, a 25 percent minority—initial mixtures ranging from 25 percent to 75 percent will survive but initial mixtures more extreme than that will lose their minority members and become all of one color. And if those who leave move to where they constitute a majority, they will increase the majority there and may cause the other color to evacuate.

Evidently if there are lower limits to the minority status that either color can tolerate, and if complete segregation obtains initially, no individual will move to an area dominated by the other color. Complete segregation is then a stable equilibrium.

Sorting and Scrambling

Minor-league players at Dodgertown—the place where Dodger-affiliated clubs train in the spring—are served cafeteria-style. "A boy takes the first seat available," according to the general manager. "This has been done deliberately. If a white boy doesn't want to eat with a colored boy, he can go out and buy his own food. We haven't had any trouble."[8]

Major-league players are not assigned seats in their dining hall; and though mixed tables are not rare, they are not the rule either. If we suppose that major- and minor-league racial attitudes are not strikingly different, we may conclude that

[8] Charles Maher, "The Negro Athlete in America," *The Los Angeles Times* Sports Section, March 29, 1968.

racial preference in the dining hall is positive but less than the price of the nearest meal.

Actually, though, there is an alternative: whites and blacks in like-colored clusters can enter the line together and, once they have their trays, innocently take the next seats alongside each other. Evidently they don't. If they did, some scrambling system would have had to be invented. Maybe we conclude, then, that the racial preferences, though enough to make separate eating the general rule, are not strong enough to induce the slight trouble of picking partners before getting food. Or perhaps we conclude that players lack the strategic foresight to beat the cafeteria line as a seat-scrambling device.

But even a minor-league player knows how to think ahead a couple of outs in deciding whether a sacrifice fly will advance the ball team. It is hard to believe that if a couple of players wanted to sit together it would not occur to them to meet at the beginning of the line; and the principle extends easily to segregation by color.

We are left with some alternative hypotheses. One is that players are relieved to have an excuse to sit without regard to color, and cafeteria-line-scrambling eliminates an embarrassing choice. Another is that players can ignore, accept, or even prefer mixed tables but become uncomfortable or self-conscious, or think that others are uncomfortable or self-conscious, when the mixture is lopsided. Joining a table with blacks and whites is a casual thing, but being the seventh at a table with six players of opposite color imposes a threshold of self-consciousness that spoils the easy atmosphere and can lead to complete and sustained separation.

Hostesses are familiar with the problem. Men and women mix nicely at stand-up parties until, partly at random and partly because a few men or women get stuck in a specialized conversation, some clusters form that are nearly all male or all female; selective migration then leads to the cocktail-party equivalent of the Dodgertown major-league dining hall. Hostesses, too, have their equivalent of the cafeteria-line rule: they

alternate sexes at the dinner table, grasp people by the elbows and move them around the living room, or bring in coffee and make people serve themselves to disturb the pattern.

Sometimes the problem is the other way around. It is usually good to segregate smokers from non-smokers in planes and other enclosed public places; swimmers and surfers should be segregated in the interest of safety; and an attempt is made to keep slow-moving vehicles in the right-hand lane of traffic. Many of these dichotomous groupings are asymmetrical: cigar smokers are rarely bothered by people who merely breathe; the surfer dislikes having his board hit anybody in the head but there is somebody else who dislikes it much more; and the driver of a slow truck passing a slower one on a long grade is less conscious of who is behind him than the driver behind is of the truck in front. Styles of behavior differ: surfers like to be together and cluster somewhat in the absence of regulation; water-skiers prefer dispersal and are engaged in a mobile sport, and rarely reach accommodation with swimmers on how to share the water.

These several processes of separation, segregation, sharing, mixing, dispersal—sometimes even pursuit—have a feature in common. The consequences are aggregate but the decisions are exceedingly individual. The swimmer who avoids the part of the beach where the surfers are clustered, and the surfer who congregates where the surfboards are, are reacting individually to an environment that consists mainly of other individuals who are reacting likewise. The results can be unintended, even unnoticed. Non-smokers may concentrate in the least smoky railroad car; as that car becomes crowded, smokers, choosing less crowded cars, find themselves among smokers, whether they notice it or not, and less densely crowded, whether they appreciate it or not.

The more crucial phenomena are of course residential decisions and others, like occupational choice, inter-city migration, school- and church-population, where the separating and mixing involve lasting associations that matter. The minor-league players who eat lunch at Dodgertown have no cafeteria-line–mechanism to scramble their home addresses; and even if they were located at random, they would usually not be casually integrated, because mixed residential areas are few and the choice, for a black or for a white, is between living among blacks or living among whites—unless even that choice is restricted.

It is not easy to tell from the aggregate phenomenon just what the motives are behind the individual decisions, or how strong they are. The smoker on an airplane may not know that the person in front of him is sensitive to tobacco smoke; the water-skier might be willing to stay four hundred yards offshore if doing so didn't just leave a preferred strip to other skiers. The clustered men and women at that cocktail party may be bored and wish the hostess could shake things up, but without organization no one can do any good by himself. And people who are happy to work where English and French are both spoken may find it uncomfortable if their own language falls to extreme minority status; and by withdrawing they only aggravate the situation that induced them to withdraw.

People who have to choose between polarized extremes—a white neighborhood or a black, a French-speaking club or one where English alone is spoken, a school with few whites or one with few blacks—will often choose in the way that reinforces the polarization. Doing so is no evidence that they prefer segregation, only that, if segregation exists and they have to choose between exclusive association, people elect like rather than unlike environments.

The dynamics are not always transparent. There are chain reactions, exaggerated perceptions, lagged responses, speculation on the future, and organized efforts that may succeed or fail. Three people of a particular group may break leases and move out of an apartment without being noticed, but if they do it the same week somebody will notice and comment. Other residents are then alerted to whether the whites or the blacks or the elderly, or the families with children or the families

without, are moving away, thereby generating the situation of minority status they thought they foresaw.

Some of the processes may be passive, systemic, unmotivated but nevertheless biased. If job vacancies are filled by word of mouth or apartments go to people who have acquaintances in the building, or if boys can marry only girls they know and can know only girls who speak their language, a biased communication system will preserve and enhance the prevailing homogeneities.

A Self-Forming Neighborhood Model

Some vivid dynamics can be generated by any reader with a half-hour to spare, a roll of pennies and a roll of dimes, a tabletop, a large sheet of paper, a spirit of scientific inquiry, or, lacking that spirit, a fondness for games.

Get a roll of pennies, a roll of dimes, a ruled sheet of paper divided into one-inch squares, preferably at least the size of a checkerboard (sixty-four squares in eight rows and eight columns) and find some device for selecting squares at random. We place dimes and pennies on some of the squares, and suppose them to represent the members of two homogeneous groups—men and women, blacks and whites, French-speaking and English-speaking, officers and enlisted men, students and faculty, surfers and swimmers, the well dressed and the poorly dressed, or any other dichotomy that is exhaustive and recognizable. We can spread them at random or put them in contrived patterns. We can use equal numbers of dimes and pennies or let one be a minority. And we can stipulate various rules for individual decision.

For example, we can postulate that every dime wants at least half its neighbors to be dimes, every penny wants a third of its neighbors to be pennies, and any dime or penny whose immediate neighborhood does not meet these conditions gets up and moves. Then by inspection we locate the ones that are due to move, move them, keep on moving them if necessary and, when everybody on the board has settled down, look to see what pattern has emerged. (If the situation never "settles down," we look to see what kind of endless turbulence or cyclical activity our postulates have generated.)

Define each individual's neighborhood as the eight squares surrounding him; he is the center of a 3-by-3 neighborhood. He is content or discontent with his neighborhood according to the colors of the occupants of those eight surrounding squares, some of which may be empty. We furthermore suppose that, if he is discontent with the color of his own neighborhood, he moves to the nearest empty square that meets his demands.

As to the order of moves, we can begin with the discontents nearest the center of the board and let them move first, or start in the upper left and sweep downward to the right, or let the dimes move first and then the pennies; it usually turns out that the precise order is not crucial to the outcome.

Then we choose an overall ratio of pennies to dimes, the two colors being about equal or one of them being a "minority." There are two different ways we can distribute the dimes and the pennies. We can put them in some prescribed pattern that we want to test, or we can spread them at random.

Start with equal numbers of dimes and pennies and suppose that the demands of both are "moderate"—each wants something more than one-third of his neighbors to be like himself. The number of neighbors that a coin can have will be anywhere from zero to eight. We make the following specifications. If a person has one neighbor, he must be the same color; of two neighbors, one must be his color; of three, four, or five neighbors, two must be his color; and of six, seven, or eight neighbors, he wants at least three.

It is possible to form a pattern that is regularly "integrated" that satisfies everybody. An alternating pattern does it (Figure 3), on condition that we take care of the corners.

	#	O	#	O	#	O	
#	O	#	O	#	O	#	O
O	#	O	#	O	#	O	#
#	O	#	O	#	O	#	O
O	#	O	#	O	#	O	#
#	O	#	O	#	O	#	O
O	#	O	#	O	#	O	#
	O	#	O	#	O	#	

Figure 3

No one can move, except to a corner, because there are no other vacant cells; but no one wants to move. We now mix them up a little, and in the process empty some cells to make movement feasible.

There are 60 coins on the board. We remove 20, using a table of random digits; we then pick 5 empty squares at random and replace a dime or a penny with a 50-50 chance. The result is a board with 64 cells, 45 occupied and 19 blank. Forty individuals are just where they were before we removed 20 neighbors and added 5 new ones. The left side of Figure 4 shows one such result, generated by exactly this process. The #'s are dimes and the *O*'s are pennies; alternatively, the #'s speak French and the *O*'s speak English, the #'s are black and the *O*'s are white, the #'s are boys and the *O*'s are girls, or whatever you please.

–	#	–	#	O	#	–	O		–	–	–	#	–	#	–	–
#	#	#	O	–	O	#	O		–	–	–	–	–	–	–	–
–	#	O	–	–	#	O	#		–	–	–	–	–	–	–	–
–	O	#	O	#	O	#	O		–	–	#	–	#	–	#	–
O	O	O	#	O	O	O	–		–	–	–	–	–	–	–	–
#	–	#	#	#	–	–	O		#	–	–	–	–	–	–	–
–	#	O	#	O	#	O	–		–	–	O	–	O	–	O	–
–	O	–	O	–	–	#	–		–	–	–	–	–	–	–	–

Figure 4

The right side of Figure 4 identifies the individuals who are not content with their neighborhoods. Six #'s and three *O*'s want to move; the rest are content as things stand. The pattern is still "integrated"; even the discontent are not without some neighbors like themselves, and few among the content are without neighbors of opposite color. The general pattern is not strongly segregated in appearance. One is hard-put to block out #-neighborhoods or *O*-neighborhoods at this stage. The problem is to satisfy a fraction, 9 of 45, among the #'s and *O*'s by letting them move somewhere among the 19 blank cells.

Anybody who moves leaves a blank cell that somebody can move into. Also, anybody who moves leaves behind a neighbor or two of his own color; and when he leaves a neighbor, his neighbor loses a neighbor and may become discontent. Anyone who moves gains neighbors like himself, adding a neighbor like them to their neighborhood but also adding one of opposite color to the unlike neighbors he acquires.

I cannot too strongly urge you to get the dimes and pennies and do it yourself. I can show you an outcome or two. A computer can do it for you a hundred times, testing variations in neighborhood demands, overall ratios, sizes of neighborhoods, and so forth. But there is nothing like tracing it through for yourself and seeing the thing work itself out. In an hour you can do it several times and experiment with different rules of behavior, sizes and shapes of boards, and (if you turn some of the coins heads and some tails) subgroups of dimes and pennies that make different demands on the color compositions of their neighborhoods.

Chain Reaction

What is instructive about the experiment is the "unraveling" process. Everybody who selects a new environment affects the environments of those he leaves and those he moves among. There is a chain reaction. It may be quickly damped, with little motion, or it may go on and on and on with striking results. (The results of course are only suggestive, because few of us live in square cells on a checkerboard.)

One outcome for the situation depicted in Figure 4 is shown in Figure 5. It is "one outcome" because I have not explained exactly the order in which individuals moved. If the reader reproduces the experiment himself, he will get a slightly different configuration, but the general pattern will not be much different. Figure 6 is a replay from Figure 4, the only difference from Figure 5 being in the order of moves. It takes a few minutes to do the experiment again, and one quickly gets an impression of the kind of outcome to expect. Changing the neighborhood demands, or using twice as many dimes as pennies, will drastically affect the results; but for any given set of numbers and demands, the results are fairly stable.

All the people are content in Figures 5 and 6. And they are more segregated. This is more than just a visual impression: we can make a few comparisons. In Figure 4 the *O*'s altogether had as many *O*'s for neighbors as they had #'s; some had more or less than the average, and 3 were discontent. For the #'s the ratio of #-neighbors to *O*-neighbors was 1:1, with a little colony of #'s in the upper left corner and 6 widely distributed discontents. After sorting themselves out in Figure 5, the average ratio of like to unlike neighbors for #'s and *O*'s

```
  # #   O # #
# # # O O O # #
# # O O     O #
# O   O   O O O
O O O # O O O
  O # # # O O O
    # # # #
O O         #
```

Figure 5

```
  # # # O     O
# # # O   O   O
# # O       O
  O   O   O   O
O O O # O O O
    # # # O O O
O # # # # # # #
O O       # # #
```

Figure 6

together was 2.3:1, more than double the original ratio. And it is about triple the ratio that any individual demanded! Figure 6 is even more extreme. The ratio of like to unlike neighbors is 2.8:1, nearly triple the starting ratio and four times the minimum demanded.

Another comparison is the number who had no opposite neighbors in Figure 4. Three were in that condition before people started moving; in Figure 5 there are 8 without neighbors of opposite color, and in Figure 6 there are 14.

What can we conclude from an exercise like this? We may at least be able to disprove a few notions that are themselves based on reasoning no more complicated than the checkerboard. Propositions beginning with "It stands to reason that . . ." can sometimes be discredited by exceedingly simple demonstrations that, though perhaps true, they do not exactly "stand to reason." We can at least persuade ourselves that certain mechanisms could work, and that observable aggregate phenomena could be compatible with types of "molecular movement" that do not closely resemble the aggregate outcomes that they determine.

There may be a few surprises. What happens if we raise the demands of one color and lower the demands of the other? Figure 7 shows typical results. Here we increased by one the number of like neighbors that a # demanded and decreased

```
#  #  #  #  O        O
#  #  #  #  O  O     O
#  #  #  #        O
   O  #  O  O  O     O
O  O  O  #  O  O  O
         #  #  O
      O  #  #  #  O
   O     O  #  #  #
```

Figure 7

by one the number that an *O* demanded, as compared with Figures 5 and 6. By most measures, "segregation" is about the same as in Figures 5 and 6. The difference is in population densities: the *O*'s are spread out all over their territory, while the #'s are packed in tight. The reader will discover, if he actually gets those pennies and dimes and tries it for himself, that something similar would happen if the demands of the two colors were equal but one color outnumbered the other by two or three to one. The minority then tends to be noticeably more tightly packed. Perhaps from Figure 7 we could conclude that if surfers mind the presence of swimmers less than swimmers mind the presence of surfers, they will become almost completely separated, but the surfers will enjoy a greater expanse of water.

Is it "Segregated"?

The reader might try guessing what set of individual preferences led from Figure 4 to the pattern in Figure 8.

The ratio of like to unlike neighbors for all the #'s and *O*'s together is slightly more than three to one; and there are 6 *O*'s and 8 #'s that have no neighbors of opposite color. The result is evidently segregation; but, following a suggestion of my dictionary, we might say that the process is one of *aggregation,*

```
   #  #        #  #
#  #  #     #  #  #
#  #  O  O  O  #  O
   O  O  O  O  O  O  O
O  O  O  #  O  O  O
   O  #  #  #  O  O  O
   #  #  #     O  O
   #  #
```

Figure 8

because the rules of behavior ascribed both to #'s and to *O*'s in Figure 8 were simply that each would move to acquire three neighbors of like color irrespective of the presence or absence of neighbors of opposite color. As an individual motivation, this is quite different from the one that formed the patterns in Figures 5 and 6. But in the aggregate it may be hard to discern which motivation underlies the pattern, and the process, of segregated residence. And it matters!

The first impact of a display like this on a reader may be—unless he finds it irrelevant—discouragement. A moderate urge to avoid small-minority status may cause a nearly integrated pattern to unravel, and highly segregated neighborhoods to form. Even a deliberately arranged viable pattern, as in Figure 3, when buffeted by a little random motion, proves unstable and gives way to the separate neighborhoods of Figures 5 through 8. These then prove to be fairly immune to continued random turnover.

For those who deplore segregation, however, and especially for those who deplore more segregation than people were seeking when they collectively segregated themselves, there may be a note of hope. The underlying motivation can be far less extreme than the observable patterns of separation. What it takes to keep things from unraveling is to be learned from Figure 4; the later figures indicate only how hard it may be to restore such "integration" as would satisfy the individuals, once the process of separation has stabilized. In Figure 4 only 9 of the 45 individuals are motivated to move, and if we could persuade them to stay everybody else would be all right. Indeed, the reader might exercise his own ingenuity to discover how few individuals would need to be invited into Figure 4 from outside, or how few individuals would need to be relocated in Figure 4, to keep anybody from wanting to move. If two lonely #'s join a third lonely #, none of them is lonely anymore, but the first will not move to the second unless assured that the third will arrive, and without some concert or regulation, each will go join some larger cluster, perhaps abandoning some nearby lonely neighbor in the process and surely helping to outnumber the opposite color at their points of arrival.

The Bounded-Neighborhood Model

Turn now to a different model, and change the definition of "neighborhood." Instead of everyone's defining his neighborhood by reference to his own location, there is a common definition of the neighborhood and its boundaries. A person is either inside it or outside. Everyone is concerned about the color *ratio* within the neighborhood but not with the arrangement of colors within the neighborhood. "Residence" can therefore just as well be interpreted as membership or participation in a job, office, university, church, voting bloc, restaurant, or hospital.

In this model there is one particular area that everybody, black or white, prefers to its alternatives. He will live in it unless the percentage of residents of opposite color exceeds some limit. Each person, black or white, has his own limit. ("Tolerance," I shall occasionally call it.) If a person's limit is exceeded in this area he will go someplace else—a place, presumably, where his own color predominates or where color does not matter.

"Tolerance," it should be noticed, is a *comparative* measure. And it is specific to this location. Whites who appear, in this location, to be less tolerant of *blacks* than other whites may be merely more tolerant of the alternative *locations*.

Evidently the limiting ratios must be compatible for some blacks and some whites—as percentages they must add to at least 100—or no contented mixture of any whites and blacks is possible. Evidently, too, if nobody can tolerate extreme ratios, an area initially occupied by one color alone would remain so.

There may be some number among the other color that, if concerted entry were achieved, would remain; but, acting individually, nobody would be the first.

We can experiment with frequency distributions of "tolerance" to see what results they lead to. (We cannot discover realistic distributions, because they would depend on the area in question; and the area in our model has not been named.) What we can do is to look at the process by which the area becomes occupied, or remains occupied, by blacks or whites or a mixture of both, and look for some principles that relate outcomes to the tolerances, the initial occupancies, and the dynamics of movement.

We assume that all preferences go in the same direction: a person need not care, but if he does care his concern takes the form of an *upper limit* to the other color that can occur in this area without his choosing to go elsewhere. There is no lower limit: there are no minority-seeking individuals, nor any who will leave if the area is not suitably integrated. Absolute numbers do not matter, only ratios. There are no individual positions within the mix: nobody is near the center or near the boundary, nobody has a "next neighbor."

To study the dynamics we assume that people both leave and return. (This is restrictive: if the preference for this locality were due merely to the fact that some people were already here and the cost of leaving were high, that cost would not be recovered by returning.) People in the area move out if the ratio is not within their color limit; people outside move in if they see that it meets their demands.

Information is perfect: everybody knows the color ratio at the moment he makes his choice. But people do not know the intentions of others and do not project future turnover. We need, too, the somewhat plausible assumption that, of two whites dissatisfied with the ratio, the more dissatisfied leaves first—the one with the lesser tolerance. Then, the whites within the locality will always have higher tolerances than any whites outside, and similarly for blacks inside and outside. The least tolerant whites move out first, and the most tolerant move in first, and the same for blacks.

Our initial data are cumulative frequency distributions of "tolerance" of the members of each color group. We can experiment with various distributions, but for the initial experiment we use a straight line.

An Illustrative Distribution of "Tolerance"

For the whites, the horizontal axis measures the number of whites, the vertical axis the ratio of blacks to whites representing the upper limits of their tolerances. Take the total of whites to be 100. Suppose the median white will live with blacks in equal numbers, so that 50 among the 100 whites will abide a black-white ratio of 1:0 or greater. The most tolerant will accept a ratio of 2:1 (is willing to be in a one-third minority); and the least tolerant will not stay in the presence of any blacks. The cumulative distribution of tolerances for whites will then appear as in the top of Figure 9. It is a straight line from 2:0 on the vertical axis to the 100 whites on the horizontal axis who comprise the white population.

Suppose blacks have an identical distribution of tolerance for whites but the number of blacks is half the number of whites, 50.

There are at least some whites and some blacks who could contentedly coexist. Fifty of the whites would be willing to live with all the blacks, though not all 50 blacks would be willing to live with 50 whites. A mixture of 25 blacks and 25 whites could be content together. There are 10 blacks who could tolerate a ratio of 1:6 to 1, or 16 whites; and any 16 among the 80 or so whites who will tolerate a black-white ratio of 10:16 would be content to join them. To explore all the combinations that might form a contented mix, but especially to study the dynamics of entry and departure, it is useful to translate both our schedules from ratios to absolute numbers, and to put them on the same diagram.

Translation of the Schedules

This is done in the bottom of Figure 9. The curve labeled *W* is a translation of the white tolerance schedule. For each number of whites along the horizontal axis the number of blacks whose presence they will tolerate is equal to their own number times the corresponding ratio on the schedule of tolerance. Thus 50 whites can tolerate an equal number of blacks, or 50. Seventy-five can tolerate half their number, or 37.5; 25 can tolerate 1.5 times their number, or 37.5. Ninety can tolerate one-fifth of their number, or 18; 20 can tolerate 32, and so forth.

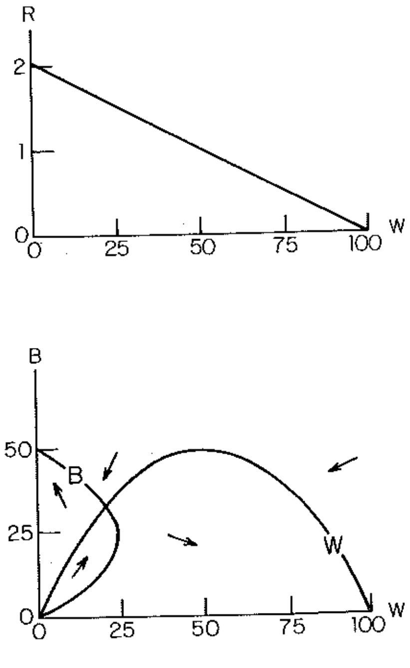

Figure 9

In this fashion the straight-line tolerance schedule translates into a parabolic curve showing the absolute numbers that correspond to the limits of tolerance of alternative numbers of whites. (Economists will recognize that the cumulative frequency distribution translates into this absolute-numbers curve in the same way that a demand curve translates into a total-revenue curve.) Similar arithmetic converts the blacks' schedule of tolerance into the smaller parabolic dish that opens toward the vertical axis in Figure 9.

Any point in Figure 9 that lies within the area of overlap denotes a combination of blacks and whites that can coexist. There are that many whites who will abide the presence of that many blacks, and there are that many blacks who will abide the presence of that many whites. Any point on the diagram that is beneath the whites' curve but to the right of the blacks' curve represents a mixture of whites and blacks such that all the whites are contented but not all the blacks. (Some of the blacks are content, but not all present.) And a point on the diagram that lies outside both curves—the region to the upper right—denotes a mixture of whites and blacks at which neither all the whites nor all the blacks could be satisfied; some of both colors would be dissatisfied.

Dynamics of Movement

It is the dynamics of motion, though, that determine what color mix will ultimately occupy the area. The simplest dynamics are as follows: if all whites in the area are content, and some outside would be content if they were inside, the former stay and the latter enter; whites continue to enter as long as all present are content, and some outside would be content if present. If not all whites present are content, some will leave; they will leave in order of their discontent, so that those remaining are the most tolerant; when their number in relation to the number of blacks is such that the whites remaining are all content, no more of them leave. A similar rule governs entry and departure of blacks.

We can now plot, for every point on the diagram, the directions of population change within this area. Within the overlapping portion of the two curves, the numbers of blacks and whites present will both be increasing. Within the white curve but outside the black curve, whites will be coming into the area and blacks departing; the direction of motion on the diagram will be toward the lower right, and nothing stops that motion until all blacks have departed and all whites have come in. To the upper left, within the black curve but beyond the white curve, blacks will be entering and whites departing; and the process can terminate only when all the whites have left and all the blacks have come in. Mixtures denoted by points outside both curves, to the upper right, will be characterized by the departure of both colors; when this movement brings one of the colors within its own curve, continued departure of the other color will improve the ratio for the color within its own curve; those who left will begin to return, and the other color will evacuate completely.

With the tolerance distributions of Figure 9, there are only two stable equilibria. One consists of all the blacks and no whites, the other all the whites and no blacks. Which of the two will occur depends on where the process starts and, perhaps, the relative speeds of white and black movement. If initially one color predominates it will move toward complete occupancy. If initially some of both are present, in "satisfied" numbers, relative speeds of black and white entry will determine which of the two eventually turns discontent and evacuates. If both are initially present in large numbers, relative speeds of exit will determine which eventually becomes content with the ratio, reverses movement, and occupies the territory.

There are, then, compatible mixes of the two colors—any mixture denoted by the overlap of the two curves. The difficulty is that any such mixture attracts outsiders, more of one color or both colors, eventually more of just one color, so that one color begins to dominate numerically. A few individuals of the opposite color then leave; as they do, they further reduce the numerical status of those of their own color who stay behind. A few more are dissatisfied, and they leave; the minority becomes even smaller, and cumulatively the process causes evacuation of them all.

Alternative Schedules

This, of course, is not the only possible result. The outcome depends on the shapes we attribute to the tolerance schedules and to the sizes of the white-black populations. The result we just reached does not depend on the fewness of blacks relative to whites: make the blacks' curve the same size as the whites' and the result is still a one-color equilibrium. But with steeper straightline schedules and equal numbers of blacks and whites, we can produce a stable mixture with a large number of blacks and whites.

Specifically, suppose that the median white can tolerate a ratio of 2.5 blacks per white, i.e., will inhabit this area even if whites are a minority of approximately 30 percent. Suppose the most tolerant can accept five to one and the least tolerant will not stay with any blacks. The tolerance schedule is a straight line with a vertical intercept at 5.0. If the blacks are equal in number and have an identical distribution of tolerance, the two schedules will translate into identical parabolas as shown in Figure 10.

Here, in addition to the two stable equilibria at 100 blacks and no whites and at 100 whites and no blacks, there is a stable mixture at 80 blacks and 80 whites. In fact, over a wide range of initial occupancies it is this mixed equilibrium that will be approached through the movement of blacks and whites. As long as half or more of both colors are present—actually, slightly over 40 percent of both colors—the dynamics of entry and departure will lead to the stable mixture of 80 blacks

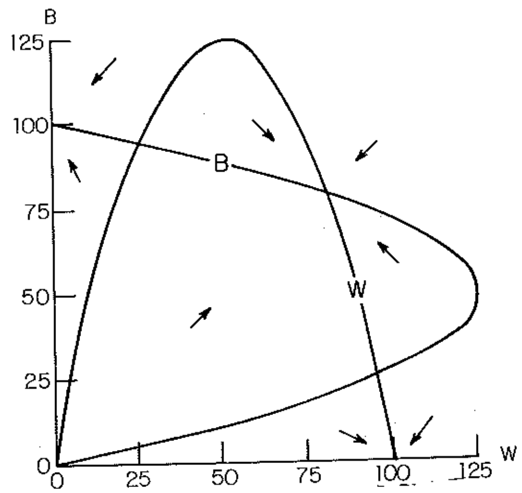

Figure 10

and 80 whites. Even for very small numbers of both colors present, if the initial ratios are within the slopes of the two curves (which allow somewhat more than four to one of either color) and if neither color tends to enter much more rapidly than the other, the two colors will converge on the 80–80 mixture. Still, if the area were initially occupied by either color, it would require the concerted entry of more than 25 percent of the other color to lead to this stable mixture. Thus each of the three equilibria—the all-white, the all-black, and the 80–80 mixture—is stable against fairly large perturbations.

Alternative Numbers

The mixed stable equilibrium generated in Figure 10 disappears if blacks exceed whites or whites exceed blacks by, say, two to one. In that case, one curve lies within the other curve, rather than intersecting it, as shown in Figure 11.

Restricting entry can sometimes produce a stable mixture. If the whites in the area are limited to 40 and if the most toler-

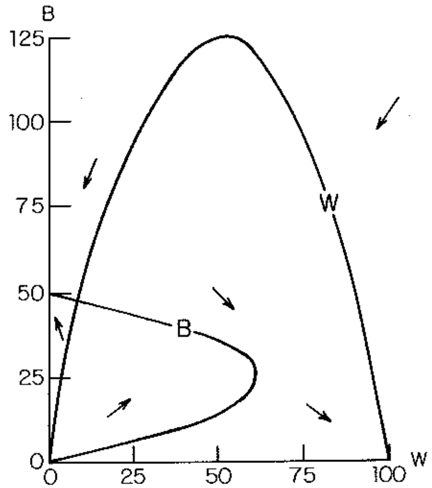

Figure 11

ant 40 are always the first to enter and the last to leave, the curves of Figure 11 are replaced by those of Figure 12, with a stable mixture at 40 whites and a comparable number of blacks. With the curves of Figure 9, however, both colors would have to be limited to yield a stable mixture.

Notice that limiting the number of whites has the same effect as if the whites in excess of that number had no tolerance at all. Whether they are excluded, or exclude themselves, it is their *absence* that keeps the whites from overwhelming the blacks by their numbers, and makes the stable mixture possible.

Thus it is not the case that "greater tolerance" always increases the likelihood of a stable mixture—not if "greater tolerance" means only that within a given population some members are statistically replaced by others more tolerant. On the contrary, replacing the two-thirds least tolerant whites in Figure 11 by even less tolerant whites keeps the whites from overwhelming the blacks by their numbers. (This would not happen if we made *all* whites less tolerant.)

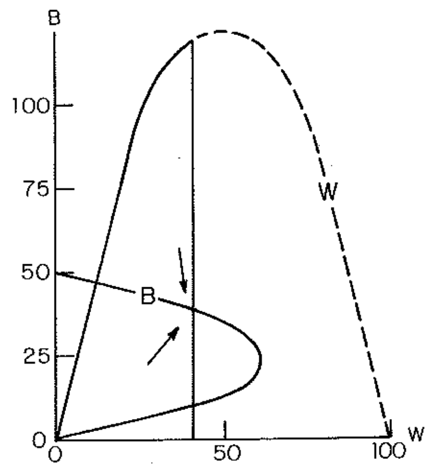

Figure 12

Varieties of Results

Evidently there is a wide variety of shapes of tolerance schedules that we could experiment with, and different ratios of blacks and whites. There is no room here for a large number of combinations, but the method is easy and the reader can pursue by himself the cases that most interest him. (The only logical restriction on the shape of the absolute-numbers curves is that a straight line from the origin intersect such a curve only once.)

Integrationist Preferences

Surprisingly, the results generated by this analysis do not depend upon each color's having a preference for living separately. They do not even depend on a preference for being in the majority!

For easy exposition it has been supposed that each person is limited in his "tolerance" for the other color and will go elsewhere if the ratio becomes too extreme. The question now arises, suppose these blacks and whites actually prefer mixed neighborhoods: what must we do to capture this neighborhood preference in a model of the sort already developed?

On reflection it appears that the analysis is already done. The same model represents both hypotheses. More than that, the same results flow from the two alternative hypotheses.

We postulate a preference for mixed living and simply reinterpret the same schedules of tolerance to denote the upper limits to the ratios at which people's preference for integrated residence is outweighed by their extreme minority status (or by their inadequate-majority status).

The same model fits both interpretations. The results are as pertinent to the study of preferences for integration as to the study of preferences for separation. (The only asymmetry is that we did not postulate a lower limit to the acceptable proportion of opposite color, i.e., an upper limit to the proportion of like color in the neighborhood.)

Policies and Instruments

The analysis is pertinent to the study of the way that numerical or ratio quotas or limits on numbers may affect the likelihood of a mixed stable equilibrium. It is equally pertinent to the study of concerted action. The occurrence of an intersection of the two curves that could constitute a stable equilibrium does not usually guarantee that that equilibrium will result. It usually competes with extreme mono-colored stable equilibria. When there are two or more potential stable equilibria, initial occupancies and rates of movement determine which one will result.

Getting "over the hump" from one stable equilibrium to another often requires either a large perturbation or concerted action. Acting in concert, people can achieve an alternative equilibrium. (Blacks and whites cannot both successfully concert in opposition to each other; either color, by concerted action, may overwhelm the other, but not both simultaneously.)

The model as described is limited in the phenomena it can handle because it makes no allowance for speculative behavior, for time lags in behavior, for organized action, or for misperception. It also involves a single area rather than many areas simultaneously affected. But it can be built on to accommodate some of those enrichments.[9]

[9] The analysis is pursued at greater length and in greater variety in Schelling, "Dynamic Models of Segregation," *Journal of Mathematical Sociology*, 1 (1971), 143–86.

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SORTING AND MIXING: AGE AND INCOME